C. LOVATT EVANS,

COLOUR VISION

TO

J. I. P.

COLOUR VISION

A DISCUSSION OF THE LEADING PHENOMENA AND THEIR PHYSICAL LAWS

BY

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IN TRIBUTE TO

PETER GUTHRIE TAIT

WHO FIRST SHOWED ME THE MAGNITUDE

OF THE SCIENTIFIC WORK

OF HIS FRIEND

HERMANN VON HELMHOLTZ

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PREFACE

More than a century ago the trichromatic theory of colour vision, foreshadowed earlier by the clear insight of Newton, took definite shape through the perceptive power of Thomas Young, "that investigator, worthy of wonder, leaping before his time." Throughout nearly half a century thereafter it remained undeveloped, until, in the strong hands of Helmholtz, it took definite mathematical shape, and was pushed to wide developments. No more self-contained, complete, and instructive example of the application of the strictly scientific procedure of Mathematical Physics could be chosen for special study. Upon the experimental basis supplied by Newton, Maxwell, and Helmholtz himself, the Newtonian law of colour mixture was qualitatively and quantitatively established. The clear perception, which saw through the meaning of apparent limitations or exceptions, was undeterred by them from acceptance of that law as the basic fact of all colour sensation. Subsidiary hypotheses were either supported by results of observation, or were specially characterized as non-essential and illustrative only. Every step taken was the simplest and most obvious step that could be taken; and it was either justified, or was replaced by the next simplest if it were found to be too limited.

Yet criticisms which have commanded some acceptance have been seemingly framed on the assumption that inessential illustrations constituted the Young-Helmholtz theory itself. And others, dealing with the phenomena of colour blindness, are apparently made in ignorance of the scope of the generalization of Young's suggestion which was made by Helmholtz. This is no doubt due in part to the fact that no translation, or even adequate account, of Helmholtz's epoch-making work had appeared in English. But it is also, and probably in most

part, due to the fact that many of those who are interested in the subject of colour vision are unacquainted with the use of the mathematical tools, simple though these be.

It cannot be too clearly realized that an adequate physical theory involves, and has as its direct aim, the establishment of a logically connected set of formal laws from the interaction of which definite results can be predicted, and these results must either coincide with previously known phenomena or with subsequently verified phenomena. In complicated cases these predictions cannot be made without the aid of mathematical analysis. Sound theory proceeds from the least complicated to the most complicated things. It is based on the most simple and most universal conditions connected with its subject. A more complicated basis may be chosen, and correct results may be deduced; yet, apart from certain special directions, these deductions are less simple, and the whole system is more artificial. In such cases strict science looks upon adequate simplicity as the hall-mark of truth.

An apt illustration is supplied by Hering's theory of colour pairs, which has been the chief antagonist of the Young-Helmholtz theory. Yet Helmholtz long ago showed that the two theories are identical in mathematical form. Hence any result deducible from the one is deducible from the other, except in so far as there may be a difference in subsidiary hypotheses. If any such subsidiary hypothesis were verified in connection with one of the views, the corresponding modification could quite readily be made in the expression of the other. The real quarrel between the two is in respect of the mode of working of the mechanism, which is, in so far as mere formal theory goes, a subsidiary matter. The simplest formulation is always selected first by physical science, for it finds that nature works in simple modes. And the trichromatic theory forms no exception. Nature works as it predicts. The statement that a particular phenomenon cannot be explained by the theory is a most unsafe one to make; and, except in so far as it is due to mere inadvertence, would only imply inability to wield the tool. Helmholtz's work can be extended to multichromasy with entire ease; but Nature intervenes.

The theory has, unavoidably, its psychical side. A fresh physical basis was needed there, and was supplied by Fechner's

psycho-physical law connecting sensation with stimulus. Fechner had himself extended its simplest form so as to take account of the effect of the intrinsic or self light of the eye. The chief triumph of Helmholtz's genius lay in his epoch-making perception that Fechner's law could be used to make possible a simple mathematical formulation of the process of colour sensation. And his next greatest lay in his application of that generalization to the problem of the settlement of the absolute fundamental sensations, which was necessarily left indeterminate by the Newtonian law of colour mixture. The decision of the editors of the third edition of his great work on physiological optics, perhaps wise on the whole, to republish the text of the first edition, not that of the second, has unfortunately excluded that treatment.

Helmholtz also placed the treatment of after images and contrast colours on a sound basis, though he gave no formal mathematical treatment. The latter he treated from the psychical point of view—a quite legitimate procedure, which avoided postulates for which there was not at the time a sufficient observational foundation. In view of the wider experimental basis now available, it is possible to bring in a mathematical representation through employment of the integrated form of Fechner's expressions of which Helmholtz used the unintegrated form in his work on differential sensitivity and the absolute fundamentals. In this way the whole subject of contrast, mutual interactions, decaying images, oscillating images, and inhibition, comes within the range of mathematical representation.

No attempt is made here to give more than a sketch of the range and power of that one theory in mathematical physics to which any postulated mode of visual activity must conform seeing that it has passed into the final stage in which a theory becomes a wider fact than any set of facts embraced within its scope, and so ranks alongside the Molecular Theory, the Electron Theory, and the Electromagnetic Theory, as being, to at least a close first approximation, a description of one of the actual modes in which Nature moves. In Sir J. H. Parson's "Colour Vision" are given fuller data and an excellently fair account of the various hypothetical views which have been put forward. It has the advantage of being written by a

physiologist who also possesses a wide knowledge of the physics involved. If in one or two respects the views herein developed may seem to differ outwardly from his, the whole trend of his attitude towards the Young-Helmholtz theory makes it likely that no real difference exists.

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W. P.

3rd October, 1922.

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"I have no faith in speculations of these kinds unless they can be reduced to exact analysis."

GEORGE GREEN.

"In developing the consequences of any valid general principle in individual cases, one constantly comes on new and quite unexpected surprises. And as the consequences are not arbitrary, nor contingent on the caprice of the author, but develop according to their own laws, I often have the impression that it is not my own work that I am writing out, but some one else's."

HERMANN VON HELMHOLTZ.

"I seem to have been only as a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

ISAAC NEWTON.

COLOUR VISION

CHAPTER I

THE GREAT PIONEERS AND THEIR WORK

"*Nihil tetigit quod non ornavit.*"

1. **Genius and Its Work.**—The development of knowledge in any one of its branches has never shown itself to be a state of steady progress except in the sense of an average taken over a lengthy period. Twenty centuries elapsed after Hiero saw reaction drive his primitive turbine until Watt saw action lift the lid of his kettle, and nearly another century passed before the turbine competed successfully with Watt's piston. Sometimes advance ceased altogether; occasionally it was reversed by procedure in wrong directions. But Time stood by it, and it never failed in the long run. Generally a period of stagnation was ended by the effort of a great worker, a man of vision and imagination and of intuitive perception in the essentialities of problems; for genius comes by nature and not by elaboration.

Genius may make mistakes, for it is human; but the mistake of genius is often more instructive and fruitful than the success of something smaller, and the chance of the occurrence of error is lessened by the presence of power. It has many grades, from the strong pre-eminence which establishes its work for ever, and builds the great edifices, to the capacity which carves and beautifies the single stones.

These features have been exemplified no less in the theory of colour vision than in any other branch of knowledge. Indeed, if one desired to recommend to a student for his instruction the study of a short self-contained branch of physical science in its experimental foundation, its theoretical evolution, its

experimental verification, and its application to the conquest of the unknown, one could not do better than propose the subject of thermoelectricity as developed on its theoretical side by Kelvin, or the subject of colour vision as theoretically elaborated by Helmholtz.

2. **Newton.**—The subject of colour had many investigators before the time of Newton, most notably perhaps the great Leonardo. Yet these dealt with it in relation to the colour of pigments, the problem being either that of increasing the number of pigments available to painters, or that of using the smallest number which could, singly or by admixture, serve their purpose. Leonardo da Vinci himself spoke of four simple colours, the term " simple " meaning uncompoundable from others. These were red, yellow, green and blue. It is a striking fact that he, great physicist as well as great painter, always calls green a " simple colour " although he knew that it was compoundable from yellow and blue pigments. Helmholtz asks, Had he perceived that the uncompounded green is much more brilliant than the compounded ? Before Newton's time it had become usual to speak of three simple colours, red, yellow, and blue.

Helmholtz deprecates the assumption of colour-sense defects in the ancients because of their restricted colour terminology. He also deprecates the assumption that, in the terminology of colour, any indication can be found regarding the essential fundamentality of certain colours ; and he thinks that, in gradating colours, our forefathers " had to seek above all things good, well-known, and always reobservable examples of striking colours." Thus arose the ideas and corresponding terms of redness in connection with resemblance to blood, of blueness in connection with resemblance to the sky or the sea, and of greenness in connection with similarity to vegetation or growing things.

It has further been pointed out that apparent confusion in colour terminology has arisen, not necessarily from absence or weakness of colour sense, but from perception of a quality held in common, such as brightness or sparkle.

The pre-Newtonian reduction of simple colours to three was quite in consonance with scientific procedure in the investigation of the colour properties of pigments. Little, if any,

further advance was possible until the relations of colour to light were established. In this matter, on the experimental side, Newton was the great pioneer, showing how ordinary white light could be resolved by the prism into coloured constituents, and how these could again be recombined together.

He subdivided the continuously varying colours of the spectrum into the well-known characteristic seven colours, red, orange, yellow, green, blue, indigo, and violet. It was Newton's genius also which treated afresh theoretically the old subject of colour mixture and started it upon a new course of development. In doing this he used his seven spectrum colours and depicted them as spread round the circumference of a circle in a definite manner. He then stated the law of colour mixture as follows : If masses proportional to the quantities of any of these definite colours used, be supposed placed at the points on the circumference which correspond to these colours, the resultant colour is represented in the diagram by the point which coincides with the centre of gravity of these masses. This law was framed to represent the previous experimental knowledge, and Newton showed that it gave good *qualitative* correspondence to the results obtained by the use of pigments. He made no reference to the system of three fundamental colours, but, as Helmholtz remarks, " he himself nowhere asserts the insufficiency of the latter system." The omissions of genius are as instructive as its assertions.

The words of Newton himself are : " I could never yet by mixing only two primary colours produce a perfect white. Whether it may be compounded of a mixture of three . . . I do not know, but of four or five I do not much question but it may." He was very obviously searching for the simplest basis.

Young himself remarks that " the optical observations of Newton are yet (a century later) unrivalled ; and excepting some casual inaccuracies, they only rise in our estimation, as we compare them with later attempts to improve on them."

3. **Helmholtz. Maxwell.**—During the period of from about one to two centuries afterwards various investigators tested Newton's law *on the basis of three fundamentals*, and gave qualitative proof of its general accuracy. In the earlier

of these tests the process of mixture of pigments was used, in the latter the correct process of mixing the lights proceeding from pigments was employed, and the results of the two did not agree.

Later, Helmholtz, working with spectrum colours, observed the discrepancy between the results obtained by the mixture of pigments and the mixture of coloured lights respectively, and first gave the true and simple explanation of that discrepancy. In the mixture of pigments the light which reaches the eye from the mixture is not in general the sum of the lights which reach it from the separate substances. It is in general so only in those cases in which neither pigment absorbs any of the light which is unabsorbed by the other.

Finally, Maxwell, working with mingled lights from separate pigments, and also with spectrum colours, gave quantitative *proof sufficient to establish the accuracy of Newton's law of colour mixture at least as a close first approximation, and that with the employment of three fundamentals only.*

4. **Young. Helmholtz.**—Before a considerable part of the experimental work referred to in the preceding section was made, Thomas Young, physiologist and physicist, had outlined a theory of colour vision giving expression to the minimum multiplicity observed to be inherent in it. That theory remained unnoticed and unappreciated until the attention of the scientific world was recalled to it by Maxwell and Helmholtz independently. It was then developed mathematically by Helmholtz, so that it is now known as the Young-Helmholtz theory. And a wider experimental basis was worked out for that development, along with searching experimental tests, by Helmholtz and members of his school.

5. **The Pioneer Work.**—The four names selected above are the names of the men who originated the chief advances in the subject, and they are all names known to fame in many other connections. They stand amongst the few names which from one century go down to succeeding centuries as those of men who have carved the original paths. "A man was famous according as he had lifted up axes against the thick trees" in the clearing of ways. These were men who designed and built and beautified the great edifices in physical science.

Newton, whose great work is only beginning, after two

centuries, to require small extension, not unforeshadowed by himself in some manner, in order to fit it for progress through widened fields in farther ages. Maxwell, whose penetrative genius was unexcelled. Young, founder of great things, whom Helmholtz himself denoted as "this investigator, worthy of wonder, leaping before his time." Helmholtz, leader in physiology, leader in physics, leader in mathematics; also an investigator, worthy of wonder, leaping, in every branch of knowledge which he pursued, before his time.

Just as in knowledge itself no claim can be laid to the absolute, so, in the work of men even of that rank, no claim can be laid to entire freedom from error. But, as above remarked, the chance is relatively small that error will be fundamental.

At a scientific meeting, when the question of the foundations of the kinetic theory were under discussion, one of the audience raised objections, and, being referred to Newton, was greatly indignant, saying that he refused to accept any authority however great. At the end of the discussion Kelvin, who was in the chair, remarked with a quiet smile, "I do not doubt that if Newton were here present, in this very room, at this very moment, he would be found to be perfectly able to take care of himself."

6. **Science.**—No system of disconnected facts can constitute a science. It is essentially a body of correlated knowledge. Therefore there can be no science unless our universe is transfused with relationships, regulations, rules, laws, essential sequences,—whatever name we give to them. In the absence of law the universe would be a chaos.

7. **The Aim of Science.**—The essential preliminary to the formation of a science is the observation and accumulation of facts. The first step in the formation is the co-ordination of these facts or phenomena, the grouping of them in accordance with identities, similarities, analogies, or correspondences of some kind. The attempt is then made to find a relation amongst some or all of these, so that, from a knowledge of the existence and magnitude of one or more, the existence and magnitude of another can be determined. That is to say, the law to which they conform is sought for.

8. **The Method of Science.**—A law may be merely qualitative, or it may also be quantitative, according as the data furnish qualitative information alone or also give information which is more or less numerically precise. These data are obtained as the result either of observation or experiment. In the latter case, conditions are deliberately altered in order that the changes in the results, if any, may be observed and measured. The determination of the law is primarily a matter of guesswork. A law is assigned and tested, as to its accuracy, by means of the data. If the calculated results correspond with the observed data within the limits of accuracy imposed by the conditions of observation and measurement, the law may be accepted until improved methods indicate inadequacy.

If the range of observation covers a wider or more complicated field the relationship, or group of relationships, which is assumed is usually spoken of as a *Hypothesis*. When the fitness is complete in so far as tests can be made, and, in particular, when hitherto unknown results which can be predicted by means of the hypothesis are found on trial to actually occur, the hypothesis is called a *Theory*. And the theory becomes more and more certain the stronger are the tests of this nature which it can provide and sustain. Nevertheless, because of the limitations of conditions and of the human mind, no theory can ever be a certainty. No knowledge can ever be sure. Faith plays its part in science as in religion. The test above alluded to, the test of fitness, gives the basis for belief.

Young's statement on this matter is full of perception. "Although the invention of plausible hypotheses, independent of any connection with experimental observations, can be of very little use in the promotion of natural knowledge; yet the discovery of simple and uniform principles, by which a great number of apparently heterogeneous phenomena are reduced to coherent and universal laws, must ever be allowed to be of considerable importance towards the improvement of the human intellect; and in proportion as more and more phenomena are found to agree with any principles that are laid down, those principles must be allowed to acquire a stronger right to exchange the appellation of hypothesis for that of fundamental laws of nature."

The grounds upon which fitness is admitted, in matters of scientific theory, are Simplicity, Accuracy, and Sufficiency. A rule or postulate which is not simple, in matters which are not complicated, is not regarded as acceptable: and, even in a complicated field, the fundamental features must be simply accounted for, however manifoldly its simply-arising ramifications may extend. Inaccuracy is obviously condemnatory. A law or theory is primarily sufficient if it accounts for all the facts for the explanation or delineation of which it was framed. It would be insufficient otherwise. Over-sufficiency is guarded against by the condition of simplicity. But sufficiency implies productivity also: theory must serve as an instrument of research, predicting unknown but actually existent phenomena.

The simplest law is that of Constancy or Invariability. The next simplest is that of direct proportionality or of inverse proportionality according as the relationship concerned is a direct or an inverse one. If more than one independent variable quantity is involved in the relation, a sum of such terms expresses the simplest case of variation.

9. **Application in the Young-Helmholtz Theory.**—As already remarked in § 1, no finer example of the application of these fundamental principles of science can be found than that which is shown in the development of the Young-Helmholtz theory of colour vision. Simple, accurate, sufficient, fit, it being "a thing of beauty is a joy for ever." Many of these features have been overlooked; in part, no doubt, because no English translation of Helmholtz's great work on physiological optics has ever appeared, and in part because of the mathematical nature of the treatment.

CHAPTER II

THE PERCEPTION OF COLOURED LIGHT

10. **A Triple Problem.**—Three independent lines of investigation are essentially involved in a complete enquiry into the nature of vision. There is first the question of the processes in nature, external to the observing eye, which are the essential antecedents to vision. Then comes the question of the mechanism underlying vision, and the mode of its activity. Lastly comes the problem of the interconnection between that activity and the response of the brain. The whole problem has separately its physical, its physiological, and its psychical sides. Concerning the last little is known, but the development of the other two can proceed independently of it ; its development cannot be contradictory to the results of the others, though it may throw light upon them, for the development must be based on the information which they give. The same is true of the problem regarding structure and its inherent activities, the physical problem must first be worked out, and it is thoroughly known ; then the facts of vision and of colour vision have to be investigated as *physical* phenomena ; and that problem, as even the preceding slight sketch may exhibit, is very definite.

The body of scientific information so found, may be expressed in varied language, some forms of which may suit definite purposes more readily even than the form which is the direct embodiment of the simplest and most fundamental conclusions which it is the aim of physical science to determine. But *every form must be consistent with every other* so long as the statements are sufficiently general. The theoretical formulation at this stage is a logical embodiment of facts and their relations, from which logical consequences can be deduced—if necessary by the processes of mathematics, which is merely

a system of symbolic logic constituting a machinery for the evolution of results to which the finite unaided human intellect could not otherwise attain. Young's and Helmholtz's theorizations give a typical example.

But the working out of a theory is often made more easy by the use of subsidiary, though not essential, assumptions. These tentative assumptions may be so restrictive as to lead to results which are not found to be consistent with observation. *This forms no objection to the general theory.* It is only necessary to remove the avoidably restrictive postulate. This procedure is in strict accordance with the scientific method. It is a step in the legitimate development of theory, and is beautifully exemplified in the treatment of colour vision by Young and Helmholtz. Yet criticisms of that treatment have been made on the erroneous view that non-essential features were essential, or that one possible conclusion was the only non-contradictory conclusion possible. This has arisen entirely from the want of recognition of the immense generality of the theory.

11. **The Triple Answer.**—In the fact that, by means of no more than three fundamental colours suitably chosen, it is possible to form a match to any spectrum colour, we have evidence, sure and unescapable, of a fundamental triplicity in the nature of colour perception. We may ignore that tripleness, substituting for it a more complicated character, and that quite successfully for particular purposes perhaps; but in so doing we cannot fail to miss, in our enquiry into the mechanism and processes of vision, the most basic aspect of the whole matter. We may deal, if we desire so to do, with a dozen fundamentals, and obtain thoroughly correct results so long as no question is raised in these of necessary relations amongst the twelve. But in doing so we are certainly ignoring the fact that there *are* nine necessary relations amongst them, and that we may be compelled to take these into consideration whenever we step outside our limited region of present enquiry. In Helmholtz's own words: "That one finds three fundamental colours sufficient contains certainly the recognition of the fact that the quality of coloured light is a function of only three variables."

But this fundamental tripleness corresponds to something

real. The reality can only be one of structure or function, and difference of function involves difference of structure somewhere. The locality of the structural triplicity may, so far as a priori considerations go, be external to the eye, within the eye or the nerve system connecting it to the brain, or within the brain itself. Brewster sought for it externally to the eye in the properties of light itself. He asserted that, in each kind of light which is visible in the spectrum, that is, in visible light of any definite refractivity, there are present three distinct kinds of light, the colour of the resultant light depending on the ratios in which the three component kinds are present. Herein Brewster, as a sound physicist, clearly recognized the necessary existence of a triple structure somewhere; but he located it wrongly, for there is no evidence of such triple structure in any light. Moreover, the assumption introduced complexity, for three different kinds of light propagated with the same speed, and presumably in the same manner, were characterized also by the same refractivity. It was thus an ad hoc postulate not compelled by any other phenomena than those of colour vision, and so violated the scientific test of simplicity and inclusiveness.

If we locate the threefold analysis in the brain *alone*, we require that light of any wavelength which is visible to the eye shall, even if its intensity be fixed, produce in the brain results which are different for the different kinds of light. This involves transmission through the optic nerve system of an effect which can only depend upon the vibrational frequency in some perfectly definite way. In other words, this view implies not only a tripleness in the brain mechanism but also a special quality in the transmission of optical effects by the nerve system: and there is no evidence of the possession by nerve substance of a transmissive power of this type.

The only other independent possibility is that the triple structure is directly connected with the nerve system. But anatomical evidence of such structure is entirely wanting. In this respect no theory is superior to another. The point of importance in this connection is that *the question is not one of crucial aspect in the Young-Helmholtz or Trichromatic Theory.* It is an important question, but a subsidiary one.

The essential feature of the theory is the essential *tripleness* of the action, and in that respect the theory is an established fact. Subsidiary hypotheses are required in its various developments, but these form only tentative suggestions, and are made, as much as for any other purpose, with the object of giving a foothold for analytical development, *the results of which would be the same whatever triply-based hypothesis were adopted.* If this is not a condition apparently unknown to most critics, it is at least unrecognized by them.

12. **Modes of Expression of the Tripleness.**—It is evident from what has been said above that, three suitable colours having been chosen as fundamental, we can completely specify any given coloured light by stating the amounts of the three fundamental colours which are present in it. This is obviously the direct expression of the fact which lies at the basis of the theory of trichromasy : but it is not the only way in which the threefoldness can be expressed. Helmholtz's statement of the case is instructive.

He points out that the impression of colour quality depends on the three independent variables, Intensity, Colour Tone, and Degree of Saturation, alone. There is no other difference, though the statement may be altered in form. Thus it follows that an amount of light of any given composition can be matched by a suitable amount of white light to which is added a correspondingly appropriate amount of a highly saturated coloured light of definite colour tone. In the case of spectrum colours the latter might be light of a definite wavelength : in the case of purple colours it might be a mixture of two lights of definite wavelengths.

Helmholtz also shows that, by this law, more restriction is placed upon the number of separately possible colour impressions, even if they are still infinite in number, than would be found if each possible combination of different lights gave a distinctive colour impression. To specify the physical nature of a given light it is necessary to state the quantity of light of each wavelength which is contained in it ; and this means that the specification requires an infinite number of parameters. On the contrary, no more than three parameters are required to completely specify the colour of the light. These may be the quantity of saturated coloured light, the

quantity of white light, and the wavelength of the coloured light.

It follows at once that, instead of these three as independent variables, we can choose any other suitable set of three, and it only remains to find by experiment the relations connecting the two sets in order that we may express the value of the impression in terms of the new units. This is a mere matter of logic. Behind it, however, lies the question whether any one such set in particular plays an actual structural or functional part in connection with the mental record of the impressions. This question will be discussed later (Chapter IX). Meanwhile it is sufficient to note that the work mentioned in Chapter I, together with a large amount of later work, proves that it is possible to select three spectrum colours, and to determine the proportions of these three which are present in any objective light, provided, in some cases, that white light be added thereto. Indeed it is possible to find many such sets of three. The work also proves that, if such a fundamental set be selected, *the law of composition is the simplest possible law connecting variable quantities. This fact makes it strongly probable that the trichromatic method of analysing a compound light impression has its basis in the natural process of composition.* Simplicity brings us into contact with nature.

If we adopt Helmholtz's direct set from the preceding page we may frame a theory which can account for the phenomena of colour vision ; and it is possible that we might therefrom be led, especially in connection with the abnormal forms of colour vision, to recognize the simplicity and sufficiency of a three-colour set. It is scarcely possible to doubt that Young or Helmholtz could have taken the step, and so would have been led to its experimental verification, predicting Newton's law of colour mixture in accordance with the scientific procedure of adopting the simplest law found to be suitable. We shall see in the development of Young's theory by Helmholtz, repeated examples of the pursuit of this kind of enquiry. It is probable that the accident of the prior experimental discovery of the existence of the three fundamentals alone deprived the theory of this other triumph.

13. **Range of Vision.**—The colours of the spectrum described by Newton (§ 2) are not seen by all eyes as having

the same extension towards the ends. Extreme red light visible to some normal eyes is quite invisible to others, and extreme violet light visible to some is also invisible to others. This is quite in analogy with the case of sound in which notes of extremely high or low frequency of vibration are audible as musical notes to some ears and are not audible to others. In both cases the sense organ may, within its limits, be quite normal. It follows that the condition of the organ is not, even when there is entire general normality in respect of colour perception, uniquely the same in all cases: and this implies some structural or functional difference. There is no fixed condition common to all eyes.

14. **Variations in Sensitiveness to Colours.**—Such a state may well raise the question whether all eyes see colour in the same way. Is yellow light, for example, productive of the same sense impression in all eyes, or does light which seems yellow to one individual produce in another the impression which is produced in the first by say green, or even blue, light? We have no a priori reason for asserting that it cannot act so except in so far as the optical apparatus is presumably of the same fundamental nature in all individuals. But the above case exhibits individual differences in so far as perception and non-perception of certain colours (red and violet) are concerned. We might, therefore, quite reasonably anticipate considerable differences of colour perception. But there is no need to rely on indirect reasoning. Experiment shows that great differences exist amongst eyes which are quite normal as regards the range of colours observable in the spectrum. If solutions of cobalt and nickel salts, the one red, the other green, be mixed together, it is quite possible, though unusual, to arrange the proportions so that to one individual the colour seems to be slightly green while to another it seems to be slightly pink. The latter case may be due either to relative deficiency in the perception of green, or to greater sensitiveness towards stimulation by red light.

It is to be noted that the former individual would not call any red spectrum light green, though he might say that a yellow light was greenish when the other saw no green tinge in it. This would happen if his eye were more sensitive to green. If it were, on the other hand, less sensitive to red, he might see

as yellow a light seen as orange by the other. Both individuals might distinguish Newton's seven colours in the spectrum. And both might be equally capable of distinguishing gradations of tint within the range of each colour. Rayleigh has shown that such differences may amount to as much as 30 per cent. from the average.

15. **Normal Phenomena.**—These are best referred to observations on the spectrum of sunlight as standard, and to wavelength as discriminative of colour tone, while the intensity is of average value. In this case those eyes which are most discriminative of colour perceive Newton's series of seven colours extending consecutively from the extreme red to the extreme violet end of the spectrum. The number of separate tints into which the whole range can be divided depends not merely on the individual, but also very markedly on the method of test, on practice and other details. A. König, Helmholtz's collaborator, could distinguish about one hundred and fifty by a delicate method of direct comparison. With ordinary rough tests perhaps about twenty may be differentiated.

Facts of the type sketched in the preceding section indicate that the word "normal" means only an average of a great many cases amongst which considerable variation is evident. It might be inferred that such variations could, in extreme cases, interfere with the apparent partitioning of the spectrum into seven colour regions. This is actually the case. To some eyes the blue and violet regions seem to gradually merge into each other without the occurrence of any intermediate region to which a separate name should be given. This question will be treated more fully in Chapter V. Meanwhile it is sufficient to note that Dr. Edridge-Green groups cases of colour vision according as seven, six, five, four, three, and so on, main colour regions can be discriminated. All these cases from seven to three would be regarded as trichromatic on the Young-Helmholtz theory. Edridge-Green's classification is specially useful when the object is to discriminate the exact capabilities of an eye as regards colour perceptions. The Young-Helmholtz classification has its scientific basis in the fact that three fundamental colours are necessary and sufficient for the matching of colours by all eyes of these types. The two classifications are not at all mutually exclusive.

The linguistic aspect of the question is of very fundamental importance. Although the objective universe of colour is the same for all eyes, the evidence is quite definite that all eyes do not perceive it alike. The most accurate perception of colour and its gradation is that which belongs to the group of eyes included in the first of the above categories. Therefore the most acute use of colour names and terms is that characteristic of the first category. And training in colour terminology will, in the highest developments, be based on its results. Eyes belonging to the less perfect categories can perceive no appropriateness in some of the discriminations made by the others. This raises very necessary doubt as to the accuracy of the meanings read by individuals possessing highly discriminative eyes into statements made by those who are not so highly endowed. No high degree of certainty can be reached in such cases, though the chance of mistaken terminology being used tends to be obviated to some extent by a natural avoidance of the use of colour terms in association with which mistakes are made most readily. Comparative assurance can only be found when the differences in the character of the perceptions occur as between the two eyes of the same individual. These cases should be specially studied.

The possibility of the incidence of confusion having psychological origin appears readily if one looks at a landscape showing varied distant and near colours, first with the head erect, and then with the head inclined to one side or inverted. The colours seem to be very much more intense in the unusual position.

16. **Abnormal Phenomena.**—It would be in entire accordance with our knowledge of the practically continuous variations which are found within the range of "normal" vision, which may conveniently be described as trichromatic in the Young-Helmholtz sense, to expect that the three-colour stages would be found to merge with practical continuity into a two-colour type of vision. This is, in fact, the case. And the considerations above referred to in connection with normal vision would naturally lead us to expect that conditions of this kind would be difficult to detect by means of the colour terminology employed. This also is largely found to be the case. Different shades of the two colours really observed are,

in consequence of education in terminology, called by different colour names. It is only when colours, seen as distinct colours by the more fully functioning eye, are given a common name by the dichromatic individual that the deficiency is readily marked. In cases of dichromasy it is at least usual that one region of the spectrum is matched with grey: sometimes there are two. And the colourless region, if we choose to take "grey" or "white" as denoting absence of colour, may be very extensive.

As a very extreme condition monochromasy may be found, the term being here used to denote the fact that one "colour" is really seen, and that it may not extend throughout the whole spectrum, in which case the remainder would be matchable with grey (§ 17).

The most extreme condition is that of total colour blindness in which every spectrum colour can be matched with grey. It is not inconceivable that a case of this kind may be one of true monochromasy, the one colour extending throughout the whole spectrum; but no discrimination of the condition could be made except in the event of the deficiency occurring in one eye alone of the same individual.

All of these conditions are accountable for on the Young-Helmholtz theory.

The occurrence of an unusual double name, e.g., red-green, in the terminology of a colour-blind person has to be very carefully scrutinized (§ 44). Such names as blue-green or orange-red, which refer to normally neighbouring colours, indicate continuous transition.

17. **Black, Grey, White, Coloured.**—In its most usual meaning the term colour blindness does not imply merely the want of power to distinguish between two colours which are usually seen to be distinct; it implies essentially that some one colour at least is indistinguishable from grey. The appearance of a body in respect to its colour or want of colour is dependent entirely upon the light which it sends to the eye. Ordinary daylight is called "white" light. Its composition, as made evident by Newton's method of prismatic resolution, is subject to slight variations according as the day is clouded or unclouded, or according as it is observed in the morning or at mid-day or in the evening, and so on. Under otherwise the

same conditions it depends on the amount of blue sky which is visible, and on the general colour tone of the landscape, but these variations are of no special account for general observational purposes.

A body which sent off again, by the ordinary process of scattering, all of the light incident upon it, would be an example of a perfectly white body. No such body exists in nature. Every body absorbs some of the incident light, and therefore appears to be less bright than a perfectly white body would appear to be under the same illumination. If all kinds of light which are present in the incident white light are absorbed in practically the same proportions as those in which they are present in the incident light, the scattered light, although it is feebler than the incident light, nevertheless has the same composition; and the body is said to be a white body. At least this is so if the intensity of the scattered light is not too small. In the latter case the body may be termed "dull white," "light grey," "dark grey," and so on.

Of course the white or grey body may be seen as red or blue according as it is illuminated by red or blue light. But, in practice, it will still be recognized as, and called, a white or grey body when contrasted with other bodies, unless it is too dull. This results from the fact that it sends to the eye more light than any of the other bodies send, so that it appears to be the brightest body present. In general the brightest body is the whitest body; and so brightness, except in otherwise obvious cases, serves as a test of whiteness.

If the quality of the light of our sun altered markedly, white bodies would still be called white bodies; and, when recollections of the appearance of the former sunlight had vanished, the altered sunlight would be accepted as white in all probability. Possibly even colour terminology would not be greatly affected unless the change in the quality of the sunlight were great. How little determinative a recollection of whiteness can be is easily observable in attempts to compound a white light, without direct comparison with a standard, from various coloured lights.

A perfectly "black" body would be one which absorbs all incident light, scattering none. Like a perfectly white body,

it is actually non-existent. The closest approach to blackness is got by observation of the practically unilluminated interior of an opaque enclosure through a very small opening (see further § 31).

Physiologically, blackness implies the absence of stimulation: psychologically, the recognition that illumination is absent is itself a positive perception. Helmholtz's own discussion of these matters is illuminating.

"With reference to white it is well to observe, that we designate as white those bodies which, as completely as possible so far as our eyes can perceive it, reflect light of all kinds. Just on this account they seem, in each kind of illumination, at least as bright as, mostly brighter than, all coloured bodies. Hence the idea of *whiteness* as a property of bodies is undoubtedly settled in the perception; but it is almost independent of the ratio in which the single colours are mixed in the illuminating light, i.e., of the colour tone of the mixture. In fact in every kind of illumination we certainly discriminate white bodies as such, if it can also be the case that we regard those bodies as white, which, seen in sunlight, seem to be of weak colour similar to the previously used artificial illumination. Thus, in candle light it may happen that we regard as white yellowish papers or cloths.

"Now the sun is by far the mightiest and the most copious source of light which we know and by whose illumination we most frequently and mainly use our eyes, which also allows all differences of colour to stand out most clearly, especially on the side of the blue tones. We consider therefore as preeminently white the colour of full sunlight. Slight departures of colour of another source of light from sunlight, or the small variations in the colouring of daylight which arise from the fact that it sometimes proceeds from the sun direct, sometimes from the blue sky, sometimes from illuminated clouds, sometimes from thick grey sheets of clouds, we observe on very great attention alone, or even indeed not at all if we have not had the opportunity of seeing the different modes of illumination directly one after the other. With this also co-operates the fatigue antecedents in the eye, which we shall discuss farther below in the study of after-images (see Chapter XIII). But with strong coloured illumination the recollection

of the frequently seen sunlight in our memory is correct enough for the recognition of the actual variation of the present illumination.

"But how uncertain and ambiguous is our representation of that which we call white appears distinctly as soon as we try to produce white through mixture of the spectrum colours if at the same time every other white light is excluded. If we have not near it before the eyes a sample of the normal white of daylight, with which we can compare the mixed colour formed, we attain only to a rough and uncertain approximation to white.

"It is therefore in my opinion unjustifiable to apply to the mixing of lights and the corresponding perception of white the great accuracy which the idea of objective white, as a property of bodies, has. Of course those who do so can cite Goethe in support. As a body colour it is characterized by its luminosity, and as such one may in imaginative language, denote it as the most serene and purest light. But if we look away from the conditions of the objective source of light, there has hitherto been found no single token whereby, amongst the different gradations of whitish colour tones, one plays a specially characteristic rôle as normal white. Since, besides, the organs of animals adapt themselves to those tasks which are most frequently set them, it is not a striking thing that the colour of sunlight occupies a central, if not strictly definable, position in the colour system.

"Black is an actual perception, i.e., observation of a definite condition of our organ, even if it is brought about by the absence of all light. We differentiate the perception of black distinctly from the failure of all perception. A patch of our field of sight, from which no light falls on our eye, seems to us black. But the objects behind our back, from which also no light falls on our eye whether they be dark or bright, do not seem black to us, but all perception fails for them. With closed eyes we are very well aware that the black field of sight has a limit, it does not extend in any way behind our backs. That part of the field of sight whose light we can perceive, if such is present, appears black if it sends out no light.

"That grey is identical with feeble white, brown with feeble

yellow, red-brown with feeble red, one recognizes most easily through prismatic analysis of the light from grey, blue or red-brown bodies, with more difficulty through projection of the light having the colour and strength under consideration upon a screen, because we continually have the inclination to separate what in the colour or appearance of a body arises from the illumination and what arises from the peculiarity of the surface of the body. The investigation must on this account be so arranged that the observer is prevented from knowing if there be a special illumination present. A grey sheet of paper which lies in sunshine can seem brighter than a white one which lies in the shade, whilst yet the first seems grey and the second white; for we know that the white sheet, laid in sunshine, would be much brighter than the grey which now is there. But, if one places a grey circle on the white paper, and concentrates light on it by a condensing lens, without the white paper being simultaneously illuminated by it, one can make the grey seem whiter than the white paper, so that in this case the quality of the perception shows itself throughout as dependent on the light strength alone.

"Thus I succeeded in making homogeneous golden yellow of the spectrum appear as brown, in that I, by means of a comparison method, illuminated therewith a small rectangular field on a white unilluminated screen, and, on the contrary, near it, illuminated a larger field of the screen with brighter light. Red treated in the same way gave red-brown, green gave olive-green."

18. **Judgment of Colour.**—The above quotation just precedes the statement given in § 12. In the last sentence we have an illustration of what may be a purely psychological effect, the perception of coloured light may be influenced by means of a standard of comparison. Thus a golden light may be made to produce the impression of brown when a strong white light is thrown on a neighbouring part of the retina. Two explanations are possible. First the neighbouring retinal structures (including nerve structure) may be structurally interrelated, so that the physiological effects in one part have an influence on those in another: or, second, the interdependence lies in the brain structure. Either supposition

is equally simple: the question is merely one of fact.

The effect of the interdependence, whatever be its nature, is that the eye judges of results *relatively*. Relatively to the strong white, the golden yellow appears brown, which is exactly the result obtained by weakening the yellow light sufficiently itself without any use of a large, strongly illuminated, neighbouring area. The retina responds less to a very feeble yellow stimulation than it does to somewhat stronger yellow stimulation: the strong white stimulation of the large neighbouring area deadens the effect of the yellow stimulation just as a lessening of its own magnitude would. The action may be described as a lessening of the capacity for perception in consequence of independent stimulation. Here the independent stimulation is on a neighbouring retinal area. The same law holds in the case of independent stimuli on the same area. It holds also, with suitable modification, when the stimuli can be of opposite quality (see below).

19. **Complementary Colours.**—In ordinary vision certain pairs of spectrum colours have the property that white light results from their mixture. Such pairs are termed complementary colours. If, therefore, we adopt Newton's scheme of colour representation (§ 2) in its general nature, the spectrum colours must be spread around the circumference in such a way that the straight lines joining complementary colours all intersect in a common point corresponding to white light. Further, if we define equal amounts of complementary lights as amounts which are complementarily equivalent, the colour curve must be symmetrical about the common point. If the amounts in each pair are taken as the same in all pairs the curve will be a circle as Newton assumed. If, as is the case, there be a region of the spectrum in which no light has a complementary colour, the spectrum colours will not extend over the whole length of the curve. The ends of the spectrum colours will also be the ends of the unoccupied region of the curve diametrically opposite the part of the spectrum (in the green) which has no complementary part. Extreme red is complementary to a blue-green; extreme violet is complementary to a yellow-green.

Whether the actual conditions are such that the curve can be a circle exactly will be considered later (§ 36).

If x and y represent units of two complementary colours, while w represents a unit of white, we have

$$x+y=2w.$$

The equation asserts that x added to y (or conversely) robs it of colour. Thus complementary colours can be regarded, where colour is concerned, as of opposite quality.

CHAPTER III

THE NATURE OF THE MECHANISM AND THE FOUNDATIONS OF THE TRICHROMATIC THEORY

20. **The Unknown Part.**—In each of the three features involved in the perception of light and colour much yet remains unknown. The structure and actions involved in the transmission of light from the exterior of the eye to the retina at the back of the eye are thoroughly known. These matters are purely physical and in no way different from corresponding things in non-living matter. The exact transformation of energy which occurs when the light falls upon the retina and its associated structures is entirely unknown. The minute structures exhibited in the retina, the rod and cone layer, ganglions and cells, nerve fibres and their ramifications, are largely known. But it cannot be said that all is known. And of the mode of communication of the energy to and along the optic nerve bundle little is known, although a good deal is known regarding the physiological properties of nerve substance. Concerning the psychical aspect, the manner in which the physical or physiological stimulus gives rise to *perception*, nothing at all is known, perhaps never can be known.

With so much unknown, what certainty can we have ? We must remember (§ 8) that scientific certainty merely means strong probability. Assumptions have to be made in order that their suitability may be tested.

A. But, without knowing all about the structure and mode of action of a mechanism, we may know enough about some of the actions to enable us to recognize something fundamentally simple which is characteristic of the chief actions. This is at least an approximate law ; and its formal conse-

quences can be developed. In general, if the phenomena are extensive and complicated, the simple law may require extension to a higher approximation in order that its results may cover the whole field with sufficient accuracy. The necessity for extension in no way diminishes the value of the simple law when results of the first order of accuracy alone are required. To make these extensions is not to bolster up a defective theory. On the contrary, when the extension is simple and obvious, its adoption furnishes a characteristic example of the true scientific method. When a physical foundation for it can be recognized the extension becomes unavoidable.

B. On the other hand, the simple law may refer to a limited tract of the phenomena concerned. In such a case, if the law be established, we may, if we so choose, shut our eyes to its existence when we deal with other regions. But the law is none the less true and essential, and a sufficiently general theory must bring it into prominence. Indeed it must be basic if it deals with a fundamental region.

C. Again, the formal laws may be obeyed, and yet may not specify a particular mechanism. Thus one and the same formal law may have application to widely different branches of physics in which the acting mechanisms are utterly distinct. In this case, when the results of the law are known with reference to one of the branches the corresponding results for another branch can be at once predicted. Conversely, we can get the correct formal results by applying the law to any one of the mechanisms even though we cannot be sure that it is the one actually concerned.

Each of these three considerations is beautifully exemplified in the development of the Young-Helmholtz theory.

21. **The Known Part.**—From these considerations it is evident that any question regarding the actual mechanism may be of much less importance to the physicist than to the anatomist or physiologist. To the physicist further discovery in this direction may merely remove an ambiguity and so give more precision to the theory while in no way invalidating its general formal conclusions. On the other hand, its value may be very great to the physicist ; for the generality of a physical theory may be so wide that it gives many avenues of development, some only, or one only, of which correspond to the

actual structure. Here, however, it is very often the case that further knowledge regarding the phenomena, apart from the structure, becomes discriminative.

In Helmholtz's words: "One may content oneself with the assumption that the optic nerve is capable of different types of perception without enquiring further concerning the ground why the system of perceptions is such a one as the eye offers."

It is known that the rod and cone layer is the seat of the earliest transformations occurring in the optical processes leading to perception; and that the retinal pigment, the so-called "visual purple," is altered by the incidence of light and quite likely plays some part in the action. It is also known that the retinal nerve fibres, and their continuations in the optic nerve bundle, convey the stimuli to the brain. Beyond these facts lies the domain of hypothesis alone.

Young chose the simplest and most direct.

22. **Young's Hypothesis.**—Helmholtz's statement of the position is as follows:

"The facts, to be concluded from the law of colour mixture, that three component sensations, proceeding independently of each other, are called forth by external excitation, have their very definite and intuitive expression contained in the hypotheses which assume that these different components of sensation are excited and transmitted in different parts of the optic nerve apparatus, and then simultaneously attain to perception and so become simultaneously localized in the same part of the field of vision in so far as they originate in the same part of the retina.

"Such a theory was first put forward by Thomas Young. Its stricter development is essentially subject to the condition that the author wished to ascribe to the light perceptive nerves only those properties and capabilities which we know certainly in the case of the motor nerves of animals and men. We have much better opportunity of measurement in this case than in the case of the receptive nerves since we can with relative ease and distinctness recognize and measure the most delicate changes of their excitation and excitability through the contractions produced in muscles and their changes. Moreover, what we have already been able to ascertain regarding the

structure, the chemical condition, the excitability, conductivity, and the electric behaviour of the perceptive nerves, agrees so completely with the corresponding behaviour of the motor nerves, that fundamental differences in the nature of their activity, so far as they do not depend on other organic apparatus associated with them, on which they exert their action, are extremely improbable.

"Now in the case of the motor nerves we know only the antithesis between the conditions of rest and of activity. The nerve can be maintained for a long time in the former without notable change of substance or development of heat. Throughout this time the muscle dependent on the nerve remains slack. If one excites the nerve, heat develops in it, changes of substance and electric oscillations occur, the muscle contracts. In the excised nerve preparation the conductivity is quickly lost, apparently on account of the consumption of the chemical ingredients necessary to the activity. Under the influence of the atmospheric oxygen, or yet better, the oxygen contained in arterial blood, the excitability is slowly restored in whole or in part, and this process of restoration does not produce muscular contraction, or changes, in nerve or muscle, of the electric behaviour which coincides with activity. Also we do not know of any external means by which we can call forth so rapidly and intensively this restorative process, and let it moreover so suddenly appear and disappear, as would be necessary did this process serve as the physiological foundation of strong and precise perception.

"If we restrict our postulates in the development of the theory of colour vision to this activity assuredly belonging to the nerves, the theory of Thomas Young is thereby given in fairly strong outline.

"The perception of darkness corresponds to the condition of rest of the optic nerve. . . ."

Helmholtz specifies Young's postulates, which are those of three types of nerve fibre excitation of which causes respectively the sensation of red, green, and violet light. The extent of excitation of each set depends on the wavelength of the light, the red sensation being stimulated most by light of long wavelength, the green most by light of intermediate wavelength, and the violet most by light of short wavelength. He points

out, however, that there are phenomena which indicate that each kind of fibre is affected by all wavelengths to a greater or less extent. The excitations are said to be equal when the three kinds are so stimulated as to give the impression of white light.

He recognizes the possible objection that this presumption necessitates the existence of a greater number of fibres than would otherwise be required, but he holds that anatomical knowledge does not contradict the presumption; and he shows that it may be quite possible for nerve fibres, fewer in number than the differentiable localities of the field of sight, to give rise to the observed accuracy of vision.

23. **Helmholtz's Hypothesis.**—In continuation of the above remarks on Young's hypothesis, Helmholtz proceeds :—

"The above sketched theory of Thomas Young is, in relation to the general theory of nerve activity as it has been developed by Johannes Müller, a special consequence of the law of *specific sensations.* In accordance with its assumptions the perceptions of red, of green, and of violet are to be regarded as determined by the specific perception energy of the corresponding three nerve apparatuses. Each arbitrary kind of excitation which can generally excite the corresponding apparatus would always be able to call forth in it only its own specific perception. The basis of the special quality of these perceptions we cannot indeed seek in the retina or the condition of its fibres, but in the activity of the central part of the brain associated with them.

"I have hitherto kept the explanation of this theory relatively abstract in order to preserve it as far as possible from more far-reaching hypothetical additions. Nevertheless, for the more certain understanding of such abstractions, there is, on the other hand, great advantage in seeking to make for oneself images thereof, as concrete as possible, even if these introduce many assumptions which are not strictly necessary to the essence of the matter. In this sense I permit myself to propose the following somewhat obvious form of Young's theory. I need not indeed explain that objections to these additions do not refute the essence of Young's hypothesis.

"1. In the end organs of the optic nerve fibres there are stored up three kinds of photochemically decomposable substances

which have different susceptibility for different parts of the spectrum. The three colour values of the spectrum colours depend essentially on the photochemical reaction of these three substances towards light. There are present in the eyes of birds and reptiles nearly colourless spindles, in fact little rods, with red, and others with yellow-green, oil drops which can effect a favouring of single simple lights in the action on the posterior parts of the structure.

"2. Through decomposition of each of the light perceptive substances the nerve fibres charged therewith are thrown into the state of excitation. There is only one kind of sensation exciting activity in each nerve fibre which takes place with decomposition of the organic substance and development of heat, as we know it of the muscle nerves. These antecedents are apparently of like kind throughout in the three systems of fibres. They only work differently in the brain in so far as they are connected with differently functioning parts of the brain. The nerve fibres require here, as generally, only to play the rôle of telegraph wires flowing through which electric currents, of like kind throughout, can set loose or call forth the most different activities in the same end apparatus connected therewith. These excitations of the three fibre systems form the above specified three *elementary excitations*, on the presumption that the intensity of excitation, for which we have yet no generally valid measure, shall be taken as proportional to the intensity of the light. This does not prevent that the intensity of the elementary excitation may be any complicated function whatsoever of the consumption of substance or of the fluctuation of current in the nerve, which latter can perhaps also occasionally be employed as a measure of the excitation.

"3. In the brain the three fibre systems are in connection with three differently functioning ganglion cells, which are perhaps so placed together in space that those which correspond to the same point of the retina lie close together. This seems to result from the newer investigations on the influence of brain injuries on the field of vision."

24. **Young's Theory and the Young-Helmholtz Theory.**—From the preceding discussion it is to be concluded that Young's theory, including its logical and mathematical develop-

ment by Helmholtz, consists (1) in the recognition as a fact of the statement that all colours can be compounded of three definite and independent colours, and (2) in the recognition that this fact implies further the co-existence in the perceiving organ of three independent and mutually non-interfering activities. The step (2) is an unavoidable (though not necessarily, in its initiation, easily perceptible) consequence of (1), unless a view such as Brewster's (§ 11) were possible. This step, and this alone, constitutes Young's theory. But there has been so much want of recognition of this point that it is desirable to emphasize the preceding evidence by a further consideration of Young's and Helmholtz's own statements.

Young's aim in taking the step may be gathered from the analogy of his incisive remarks on the theory, as distinguished from the laws, of gravitation: "Newton appears to have considered these laws of gravitation, which he first discovered, rather as derivative than as original properties of matter; and although it has often been asserted that we gain nothing by referring them to pressure or to impulse, yet it is undoubtedly advancing a step in the explanation of natural phenomena, to lessen the number of general principles; and if it were possible to refer either all attraction to a modification of repulsion, or all repulsion to a modification of attraction, we should make an improvement of the same kind as Newton made, when he reduced all the diversified motions of the heavenly bodies to the universal laws of gravitation only."

That one step constitutes also the Young-Helmholtz theory; for Helmholtz's contribution, apart from the experimental widening of the basis (1), was the logical development of the consequences of the step (2). No one could have more competently given that development than Young himself; but, at his time, the known facts did not call for its elaboration. He reduced all the diversified phenomena of the domain of colour to the universal laws of superposition of three effects.

Young's own words were the following: "Sir Isaac Newton observed that the effect of white light on the sense of sight might be imitated by a mixture of colours taken from different parts of the spectrum, notwithstanding the omission of some of the rays naturally belonging to white light. Thus, if we

intercept one half of each of the four principal portions into which the spectrum is divided, the remaining halves will still preserve, when mixed together, the appearance of whiteness; so that it is probable that the different parts of these portions of the spectrum, which appear of one colour, have precisely the same effect on the eye. It is certain that the perfect sensations of yellow and of blue are produced respectively, by mixtures of red and green and of green and violet light, and there is reason to suspect that those sensations are always compounded of the separate sensations combined; at least this supposition simplifies the theory of colours: it may, therefore, be adopted with advantage, until it be found inconsistent with any of the phenomena; and we may consider white light as composed of a mixture of red, green, and violet only, in the proportion of about two parts red, four green, and one violet, with respect to the quantity or intensity of the sensations produced."

"From three simple sensations, with their combinations, we obtain seven primitive distinctions of colours; but the different proportions in which they may be combined, afford a variety of tints beyond all calculation. The three simple sensations being red, green, and violet, the three binary combinations are yellow, consisting of red and green; crimson, of red and violet; and blue, of green and violet; and the seventh in order is white light composed of all the three united. But the blue thus produced, by combining the whole of the green and violet rays, is not the blue of the spectrum, for four parts of green and one of violet make a blue, differing very little from green; while the blue of the spectrum appears to contain as much violet as green: and it is for this reason that red and blue usually make a purple, deriving its hue from a predominance of the violet."

The fact specified in the first sentence of the statement by Helmholtz, on page 11, indicates the presence in colour perception of a manifoldness much less complicated than that associated with wavelength. Newton left its degree unsettled. Young fixed it at the minimum possible and sufficient. The point is otherwise put by Young in the course of remarks on the following queries by Newton ["Optics," Qu. 16, 13, 14]: "Considering the lastingness of the motions excited in the bottom of

the eye by light, are they not of a vibrating nature ? Do not the most refrangible rays excite the shortest vibrations,—the least refrangible the largest ? May not the harmony and discord of colours arise from the proportions of the vibrations propagated through the fibres of the optic nerve into the brain, as the harmony and discord of sounds arise from the proportions of the vibrations of the air ? "

Young says : " Since for the reason here assigned by Newton, it is probable that the motion of the retina is rather of a vibratory than of an undulatory nature, the frequency of the vibrations must be dependent on the constitution of this substance. Now as it is almost impossible to conceive each sensitive point of the retina to contain an infinite number of particles, each capable of vibrating in perfect unison with every possible undulation, it becomes necessary to suppose the number limited ; for instance, to the three principal colours, red, yellow, and blue, of which the undulations are related in magnitude nearly as the numbers 8, 7, and 6 ; and that each of the particles is capable of being put in motion less or more forcibly, by undulations differing less or more from a perfect unison ; for instance, the undulations of green light, being nearly in the ratio of $6\frac{1}{2}$, will equally affect the particles in unison with yellow and blue, and produce the same effect as a light composed of those two species : and each sensitive filament of the nerve may consist of three portions, one for each principal colour. Allowing this statement, it appears that any attempt, to produce a musical effect from colours, must be unsuccessful, or at least that nothing more than a very simple melody could be initiated by them ; for the common period, which in fact constitutes the harmony of any concord, being a multiple of the periods of the single undulations, would in this case be wholly without the limits of sympathy of the retina, and would lose its effect ; in the same manner as the harmony of a third or a fourth is destroyed, by depressing it to the lowest notes of the audible scale." He added later that " in consequence of Dr. Wollaston's correction of the description of the prismatic spectrum, compared with these observations, it becomes necessary to modify the supposition that I advanced . . . respecting the proportions of the sympathetic fibres of the retina, substituting red, green and

violet, for red, yellow, and blue, and the numbers 7, 6, and 5, for 8, 7, and 6."

The solution of the problem of colour vision seems to be extraordinarily simple when it is once found. Its real difficulty is made evident by the numerous attempts which were made unsuccessfully by physicists and physiologists alike, including Helmholtz, as he acknowledges himself, to discover it, long after the simple explanation had been given by Young. Young's work remained unnoticed until Helmholtz unearthed it nearly half a century later. And Helmholtz remarks that this was only one of many pieces of Young's work which suffered a like fate. For, as he says, though Young was looked upon with astonishment by his contemporaries, he was so much in advance of his time that his ideas were not understood, and a later generation only came slowly to the recognition of the power of his perceptions and the strength of his reasoning.

Helmholtz begs his readers to take note of the fact that the conclusions which he himself draws regarding visual sensation are not affected by any merely hypothetical matter introduced in the development of the theory. The hypothetical matter is merely illustrative and conducive to more ready realization. As a particular case he has pointed out that the idea of three separate sets of nerve fibres are not essential to Young's view. Three independent types of action alone are necessary. Their mode of emergence is a matter for discovery. Nevertheless, in spite of Helmholtz's clear warning, critics repeatedly make the mistake, and fail to comprehend aright in other directions also.

Having laid down this clear caution, Helmholtz proceeded, as has been mentioned, to point out that Young's view is only a further extension of Johannes Müller's law of specific sensations. The sensation of light or the sensation of warmth ensue in accordance with the incidence of sunlight upon nerves of sight or nerves of feeling. So Young regards difference of sensation of colour as dependent on the incidence of light upon different nerve mechanisms. Though there is no anatomical basis yet known in men and animals for Young's postulate, yet in the eyes of some birds and reptiles absorptive substances are present in the rods, and these may be efficient in determining the access of particular ranges of wavelength to

the underlying mechanism. More extended knowledge on this subject is now available, but nevertheless the anatomical and physiological conditions of vision have still to be found. Young gives the formal laws to which, whatever they may be, they must be subject.

It is to be noted in this connection that the absorptive mechanism referred to merely allows light of particular ranges of wavelength to reach definite fibres, and that it transforms the energy associated with the other wavelengths. If transmitted light alone may give rise to nerve stimulation, the question of the presence or absence of coloured absorptive media is quite a secondary one, or rather, it may be so.

25. **Helmholtz's Hypothesis.**—If, on the other hand, it be the transformed energy which is effective in nerve stimulation, we have therein the basis of Helmholtz's subsidiary hypothesis of photochemical action. The extraordinary sensitiveness of the optic nerve fibres now comes into consideration. They are "incomparably more sensitive to rays of light than any other nervous apparatus of the body, since the rest can only be affected by rays which are concentrated enough to produce noticeable elevation of temperature." Thus, and also in view of the extraordinarily great value of the energy of molecular combination, it may well be the case that the amount of the absorptive substance associated with each nerve fibre is of ultramicroscopical magnitude; or that, if it be spread throughout a visible volume, it may impart no visible tinge of colour.

Helmholtz points out that all nerves have apparently the same structure, and that the differences of effect produced by their stimulation depends upon the mechanism to which they are attached; whether, for example, they are sensitive nerves conveying activity from the brain to the muscles, or glandular nerves stimulating secretions, or cardiac nerves regulating circulation. They can all be actuated by the same excitations, mechanical, electrical, chemical, or thermometric. In each case of sensory stimulation of the brain, the impression produced is the same, for any one type of nerve, whatever be the nature of the originating stimulation. A motor and a sensory nerve, being divided, can have their halves interchanged and grown together: stimulation of the sensory nerve then causes motion. Even in a totally dark room a strong impression of

light can be produced by pressure of the finger upon the side of the eyeball. In that case there is no objective light present. Yet the sensation of light may be so strong that the light, if it were really present in the retina, could not fail to be clearly visible to another observer in the room. Here the impression of the presence of light is due to excitation of the optic nerve by an agency which is of entirely non-luminous origin. The specific sensation follows the excitation whatever may be the external agency which gives rise to its development.

The impression of colour originates in the brain after the nerve transmitted impulse reaches it. But these impressions are of multitudinous variety. Therefore the brain mechanism must be of multitudinous complexity ; or, if it be relatively simple, there must be relations amidst the varieties of impressions leaving only a finite number of these independent ; that number coinciding with the number of independent types in the brain mechanism which are efficient in the production of impressions of colour.

In defect of knowledge of the brain mechanism we can appeal to a knowledge of the least number of independent elements which are indicated amidst colour impressions. The experimental proof that in general three are necessary, and that they are sufficient, is absolutely conclusive. There must be three centres of colour impression, or three types of centre, on the one hand ; or, on the other, there may be one centre or type of centre, in which case it must be capable of three different types of stimulation.

The optic nerve is the only nerve in connection with which this tripleness of activity is evident. If, therefore, it functions just as all other nerves do, and if a threefold activity is propagated by it, any other type of nerve should give a threefold type of propagation. But it does not follow that other brain centres should respond differently to the three kinds of stimuli. Nevertheless such a case of abortive functioning in connection with the nerves concerned in all but one type of sense impression can scarcely be credited as at all probable. Indeed all presumption based on a scientific procedure is against its recognition.

But there must be a one-to-one correspondence between the eye end and the brain end of the mechanism, in regard to

the triple effect, if there is only one type of activity propagated along their connecting nerve linkage. For otherwise there would be nothing to make the transmitted impulse partition itself differently amongst the three brain centres, however much the nature of the light giving rise to the impulse might vary. Therefore the retinal mechanism must be triple.

There are two possibilities. One is that which formed Young's working hypothesis, in which it is assumed that the fibres are arranged in three sets which through connection to separate brain centres give a triple quality to the brain impression. The other possibility corresponds to the case already rejected. All fibres may be alike, but each may respond differently to three different stimuli (supplied say through the medium of three different photochemical actions originating at the retinal end). The physiological difficulties in the way of this view are probably insuperable.

26. **Conclusions.**—From all considerations, apart from the detailed investigation of the theory and its comparison with observation, the conclusion is immediate that the simplest, and therefore initially the most probable, view is that of the trichromatic theory as first formulated by Young and subsequently elaborated by Helmholtz. But Young's hypothesis regarding the mechanism and Helmholtz's extension of it, though most probable, are not essential parts of the theory, but give merely an illustrative model obeying the requisite laws.

The formal laws of action, and these alone, constitute the essence of the theory. And in so far as these formal laws correspond to the observed facts on which they are based, the theory is simply a *fact*, wider and more inclusive than the limited facts upon which it is based, having attained the goal to which all sound theories tend. On this point, as on others, Helmholtz made no mistake. He said (§ 21) that one may content oneself (in so far, that is, as the deduction of consequences and their test by experience is concerned) with the assumption that the optic nerve is capable of different types of perception without inquiring further concerning the ground why the system of perceptions is such a one as the eye presents.

27. **Helmholtz's General Discussion of Trichromasy.**—"Every kind of additive connection of any natural quantities

can be based directly on the foundation of a system of measurements of these quantities; so also the law of colour mixture. If we have tc deal with quantities having a manifoldness of one dimension, it is sufficient to choose one definite quantity of this kind as a unit of measurement. It is quite arbitrary how great we choose it. But if the quantities considered belong to a province of three dimensions, we have in general to choose three arbitrary units, not only according to quantity but also according to quality. In space measurements not only is the chosen length unit arbitrary, but also the three directions of co-ordinates to which we wish to refer all other estimates of position. Since, however, in this case we can attach the same length scale to each of the co-ordinate directions, we can at least adopt the same unit of length for all directions. In the colour system likewise we have a manifoldness of three dimensions, and must therefore choose three arbitrary units. The arbitrariness of the chosen reference colours corresponds here to the arbitrariness of the co-ordinate directions of space. We shall have to discuss in the following paragraphs the possibility of referring the quantities of the three different lights to a common fundamental measure. Here also, as with space co-ordinates, it is mainly secondary considerations, or even hypothetical views, which determine for us the choice of one or other co-ordinate system.

"If now we have chosen the ground colours and their quantitative units R, G, V, the physiological impression of any other colour F can be fully expressed by saying that it looks just like an association of so many units of R, G, and V. Thus, if we specify numbers by x, y, z,

$$F = xR + yG + zV.$$

We attain thereby just the same thing as if we specify to ourselves or others the length of a rod by expressing it in centimetres.

"But what we here compare and measure is a physiological action of light upon the eye, which, moreover, is influenced by all kinds of individual and physiological conditions, which have been already mentioned in part, and shall in part be mentioned later. Objective amounts of light come into consideration here only as excitations of sensation, and have

a measurable physical value as such. If thus, for example, we have chosen, as is yet quite arbitrary, as ground colours certain quantities of red, green and violet, with the units R, G, V, and had for the compound light F the colour equation

$$F = xR + yG + zV,$$

x would be called the red value, y the green value, z the violet value of the light.

"But it now follows therefrom further that there must be in the sensation of the eye three corresponding kinds of activity, which can exist together without mutually destroying each other, and on which depends all difference of colour sensation.

"If we assume that some procedure is found . . . for the determination of three measurable quantities ϕ, ψ, χ which together define completely the sensation of the eye, it would at any rate be possible to ascertain through observation how the values of these quantities ϕ, ψ, χ depend on the values x, y, z of the incident light. Thus ϕ, ψ, χ could at once be represented as three functions of x, y, z, as also conversely x, y, z as functions of ϕ, ψ, χ. Since no two group values of x, y, z give the same sensation, i.e., the same values of ϕ, ψ, χ, so also must x, y, z be expressible unambiguously in terms of ϕ, ψ, χ. These functions of ϕ, ψ, χ representing the values of x, y, z are thus quantities, which depend solely on the peculiarities of the sensation and are determined through the nature of the sensation, to which pertains on the other hand a certain self-continuity of existence, since each with the two others, and undisturbed by the others, can be excited in the nerve apparatus, persist, and again vanish. But it is exactly these mutually undisturbed existences which must be our object if we are to speak of *Elements*, or *Components* of the sensation. Thus if we denote the function of ϕ, ψ, χ representing x by r, and correspondingly the two others by g, v, these quantities r, g, v are in fact to be denoted as elements of the sensation. But all additive aggregates of the first order of the form $(ax+by+cz)$, where a, b, c are positive or negative numbers, would exhibit the same type of mutually undisturbed existence. Which of such linear functions of r, g, v we choose as the most suitable remains as yet undecided.

"But, on the other hand, there are none other than those linear functions of r, g, v, which, if two lights of the colour values x_1, y_1, z_1, and x_2, y_2, z_2 are superimposed, simply add themselves and can be regarded as giving the corresponding elements of the colour sensation. Naturally this does not exclude the occurrence of other actions in the range of luminous sensation which depend in a more complicated way on x, y and z. We shall become acquainted with such action in the differential sensitivity of the eye and the intensity of sensation determined in accordance therewith. But such quantities for which there is not the possibility of proving also quantitative, mutually undisturbed, existence, we shall not, if we wish to speak accurately, be able to call elements of sensation.

"Since the misunderstanding of this law has caused great confusion in colour theory, I permit myself to give here its relatively simple proof.

"The problem is whether there can be found any one function F of the magnitudes x, y, z, which, added to the same function of other values, ξ, η, ζ of the same variables gives the function of $x+\xi$, $y+\eta$, $z+\zeta$.

"This is equivalent to the question, under what conditions can an equation

$$\mathrm{F}(x+\xi,\ y+\eta,\ z+\zeta)=\mathrm{F}(x,\ y,\ z)+\mathrm{F}(\xi,\ \eta,\ \zeta)$$

subsist for arbitrarily changeable values of the variables?

"If it exists for a given value of x and for one infinitely little different therefrom, the equation when differentiated with regard to x must again give an equation which is valid for all values of the variables

$$\frac{d\mathrm{F}(x+\xi)}{d(x+\xi)}=\frac{d\mathrm{F}(x)}{dx}.$$

"The function $\mathrm{F}(\xi)$, which is independent of x, does not occur therein. If one differentiates the latter equation similarly, whether with regard to ξ or η or ζ, the second function which depends on x alone vanishes, and it follows that all second differential coefficients of the function $\mathrm{F}(x+\xi)$ taken with regard to any of its variables $x+\xi$, $y+\eta$, $z+\zeta$, are zero.

"But if simultaneously for a function of x, y, z we have

$$\frac{d^2F}{dx^2}=\frac{d^2F}{dx.dy}=\frac{d^2F}{dx.dz}=0,$$

it follows that dF/dx is independent of x, or y, or z, and so must be a constant. The same holds for dF/dy and dF/dz. Hence it follows finally in well-known manner that F can only have the form

$$F=ax+by+cz,$$

where a, b, c are independent of x, y, z.

"In all it is to be concluded from these discussions—

"1. That in any one section of the conducting nerve substance, under the influence of coloured light, three different, mutually independent, and not mutually interfering elementary activities come into play; we shall call them the *elementary excitations.* Their magnitude is proportional to the corresponding colour values x, y, z of the objective light; they correspond to the r, g, v of the above representation.

"2. That all further activities appearing in the brain, even the actual sensations attaining to consciousness in the given condition of the reacting parts of the brain, are only effects whose magnitudes are in accordance with the functions ϕ, ψ, χ of the three elementary excitations r, g, v.

"3. That either the elementary excitations themselves, or three not mutually interfering effects depending on them, are independently conducted to the central organ.

"Regarding these further activities in the deeper organs we know nothing with certainty at present. Investigation of the recognizable differences and the operations of sensation will give us sure holding points.

"Since in the whole of this domain we have always to deal alone with actions of objective light on organs of living bodies, it is self-evident that the physiological conditions of these organs, the changes in their excitability, as it is called in physiology, have influence on the magnitude and nature of the action; and so influence the interdependence of ϕ, ψ, χ on the one hand, and x, y, z on the other, so that, in the equations which express the dependence, yet other variables enter which depend on the conditions of the organs but not

on the light incident at the time. These would also enter into the values of r, g, v, expressed in terms of ϕ, ψ, χ and indicate a certain variability in the magnitude or nature of the phenomena. Changes in the magnitude are actually known, and shall be mentioned in the theory of after images.

"Further there is above all no ground there for assuming that we shall be able, by a direct act of consciousness, to separate from one another these so defined elements, so as to recognize them directly as elements. According to custom we fix our attention only on such differences of sensation as depend in regular manner on certain objective relations of surrounding Nature. The main object of our attention concerning colour is the correct estimation of body colours. In fact we attain greater certainty therein in so far as special practice or favourable conditions of observation exist for the true comprehension of the changes of body colours arising through sky-colour, illumination, or contrast. Indeed, in the domain of body colours, white has a prominent position; whitish colours are most usually seen. They form the centre of the whole colour-world; and what is not white seems only a departure from white. We judge it by the magnitude of this departure (saturation), and its direction (colour-tone). These are the relations, as already remarked, which we seek to fix in speech. Thus on the whole it is to be expected that language will seek to distinguish by special names differences which emerge almost equally clearly to immediate perception.

"It is always worthy of note that this system of names, elaborated solely to denominate those similarities or differences of colours which may be directly perceived, agrees, at least in its chief features, with the arrangement of colours represented in the colour diagram or the colour pyramid.

"Thus it becomes in some measure probable that the sensations attaining to consciousness themselves correspond to relatively little intricate or altered functions of certain elementary excitations.

"Other investigators indeed, as E. Hering and C. Donders, assume that new combinations originate here from the elementary excitations, which step into consciousness independently of one another and are discriminated as separate."

The colour diagram and colour pyramid (§ 34) referred to

above are geometrical representations of the results of the trichromatic theory. Helmholtz here points out that colour nomenclature is very closely accounted for on the basis of that theory. But the nomenclature is determined by the sensations themselves. Therefore he points out further that the sensations seem to correspond very closely in their laws of composition to the simple trichromatic laws of composition of excitations in accordance with Newton's law of colour mixture. Hering's theory, as noted in the Preface, is also trichromatic; but its subsidiary hypothesis, which replaces that of the Young-Helmholtz theory, is not so simple as that of the latter. The law of direct positive relationship between the final cause of sensation and the excitation is the simplest that could hold: and, if it holds, the condition of a direct connection between the seat of sensation and the excitation, and of some simple mode of its functioning, is clearly indicated (see § 67).

CHAPTER IV

THE SENSATIONS OF BRIGHTNESS AND COLOUR

28. **The Intensity of the Sensation. Fechner's Law.**—Since the idea of brightness is common to all estimates of the mental impression produced by lights, coloured or colourless (§ 12), it is essential to consider first the common law which connects the intensity of the sensation with the intensity of the objective light irrespective of the special nature of the light. This is the more important in that Helmholtz based his extension of the investigation, so as to include in one formula the results for all kinds of light (§ 45), upon the answer to the more simple problem.

Although (§ 8) the simplest law connecting the variations of two related quantities is that of constant proportionality, opinions might differ as to which was the next simplest. In any case, a very simple one is that of fractional proportionality, in accordance with which one quantity varies by constant amounts when a quantity upon which it depends varies by a constant fraction of its amount. In fact, this law is the law according to which capital accumulates at compound interest, while the former is that according to which capital accumulates at simple interest. And it is this somewhat more complex law which, approximately at least, regulates the dependence of the intensity of a sensation of light upon the intensity of the light itself.

If $d\mathrm{S}$ represents the change in the magnitude of the sensation S produced by a change $d\mathrm{I}$ in the intensity of the light, the law is expressed by the equation

$$d\mathrm{S} = k\frac{d\mathrm{I}}{\mathrm{I}},$$

where k is a constant.

The question of fact with which the enunciation of either of the laws above specified is concerned is this: Does the perception of the eye respond proportionally to equal increments of total stimulation or to equal increments of fractional stimulation? In the case of the estimation of the height of a tower in process of erection, the eye can measure equal increments of height and also the proportion of the increment to the total height. It could give evidence as to whether the rate of increase of the height with time followed the law of simple interest or the law of compound interest. The problem which in this case corresponds to the estimation of brightness is this: If, when the tower is 100 feet high, an increase of 10 feet is required to just give assurance that the height has actually increased, will an increase of 20 feet be similarly needed when the height is 200 feet, and so on? If so, Fechner's law correctly describes the perception. It does so describe it when the tower is sufficiently far off. At too small a distance, the entry of the law of perspective gives rise to deviation. Similarly, throughout a great range of variation of the intensity of light, proportionate increments of intensity are requisite in order that the eye should just perceive the change. When the intensity is too feeble or too great, other conditions of perception enter in, and the law ceases to apply.

29. **Weber's Law and Fechner's Law. Threshold Values.**—The first statement of the above law, dS representing the least perceptible change in sensation, was first given by Weber. In actual observation the ratio of dS to S is very small, but not excessively so. It is finite though small. If an excessively sensitive eye were found, and if the law above stated were true for its excessively small steps of sensation, it would not be true for larger steps. The general statement then would be that the change in sensation is proportional to the logarithm of the change of intensity, or that the change in sensation varies in arithmetical progression as the change in intensity varies in geometrical progression. It is this form of the law which is properly known as Fechner's Law.

The physical process involved in perception is proceeding by continuous variation as the intensity increases continuously and there is no evidence that the physiological process, and correspondingly some brain process, is not proceeding con-

tinuously also. Therefore the truly representative equation should properly be one representing the summation of infinitesimal steps and agreeing with observation when the steps become finite. Fechner's form of the law does this. It can be expressed in the form

$$I = I_0 \varepsilon^{\frac{S}{k}}.$$

Here I_0 is the "threshold value" of I, i.e., the amount which just gives rise to perception. From this we find, for two different values,

$$\frac{I_2 - I_1}{I_1} = \varepsilon^{\frac{S_2 - S_1}{k}} - 1.$$

This shows that, when the fractional change of intensity is constant, the step in sensation is constant even when finite. In that respect therefore both laws agree. The difference between them is that Weber's law makes the magnitude of the sensation proportional to the fractional change of intensity, while Fechner's law only gives that result when the steps are excessively small. In other words, the two laws give different *definitions* of the measure of a sensation. And we have no means of measuring a step in sensation apart from definition. Though we choose, on Weber's view, to say that the just perceptible steps in sensation, produced by equal fractional steps in intensity, are all equal, that is a pure assumption which we cannot directly verify. Fechner's definition, which makes the equal increments of sensation proportional to the logarithm of the corresponding step in intensity, is, so long as the intensity is neither too small nor too great, verified by experiment as a good approximation over a wide range of intensities, so that the experimental support of his definition is good.

30. **Perception of Different Kinds of Light.**—Fechner's law applies to all kinds of light, but the value of k depends on the wavelength, so that, in any one experimental test with a compound light, the proportionate representation of wavelengths must be maintained.

Fig. 1, reproduced from Helmholtz's "Physiological Optics," exhibits the region in which Fechner's law is closely obeyed and the manner in which deviations from it occur at high and

low intensities and for different wavelengths of light. The ordinates represent the values of di/i, where i is the intensity of the light, and di represents the minimum increase which can just be perceived. Visual discrimination is therefore weak when the ordinates are great, and conversely. In order that a great range of intensities shall be represented, equally spaced abscissæ represent the logarithms of the intensities instead of the intensities themselves. Thus, for example, the range of intensities included in the stretch 100 to 10,000 is 100 times as great as that included in the stretch 0 to 100.

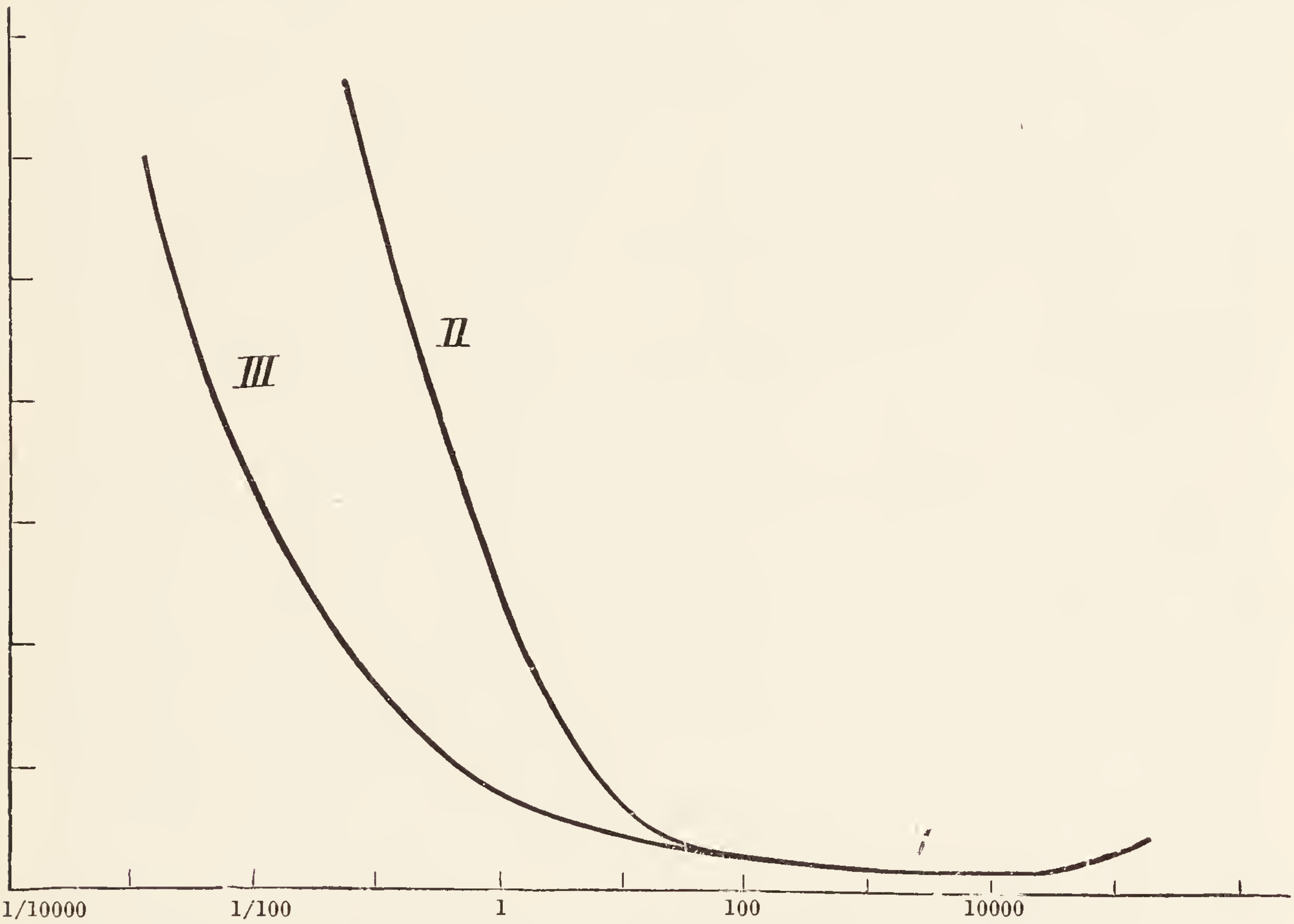

Fig. 1.—Change of Sensation with Intensity of Illumination.

If di/i is constant, Fechner's law is obeyed. The closeness with which it is followed throughout a great range of intensities, for all wavelengths, is exhibited in the region *I* of the diagram ; and the great deviations from it, in the direction of decrease of sensitiveness as the intensity weakens, are shown by the branches *II* and *III*. The former represents the effect with blue light, the latter with reddish-yellow light.

31. **Stimulation and Sensation.**—When feeble stimulation is employed, such as that afforded by stars, it is found that eye estimates of brightness do not agree with estimates deduced

by purely physical processes, such as the measurement of the energy falling per second on unit area of the retina. Fechner showed that the eye estimates were closely reconcilable with photographic measurements if the formula of § 28 were replaced by

$$dS = k\frac{dI}{I + I_0},$$

I_0 being a constant. At very small and very great intensities, the formula, though suiting better than the unmodified one, deviates considerably from observation. It follows from the expression that, in order to just excite the sensation of a difference in brightness, a larger increase in the intensity of the external light is required when the quantity I_0 is appreciable than would be needed if it were null. The magnitude of I_0 determines the insensitiveness of the eye to gradations of brightness. Fechner ascribed its presence to the influence of disturbing conditions, and Helmholtz pointed out its connection with the "self light" or inherent luminosity of the eye.

In a totally dark room the eye perceives light as an irregularly-flecked shimmer. The source of this self luminosity is doubtful, for various causes, such as pressure on the eye, give rise to the sensation of light. It may be due to the same retinal action as that which is aroused by external light. If, for example, Helmholtz's hypothesis, that light causes chemical change in some substance present in the retinal mechanism (§ 23), is correct, molecular motions will always ensure the presence of a small amount of decomposition, and presumably therefore also of nerve stimulation. Against this view there is Burch's observation that during very prolonged resting of the eye in an absolutely dark room the self light slowly diminishes and finally disappears: but the objection is not necessarily fatal to the view. And the influence of the term I_0 upon phenomena of perception can be followed out without any reference to its origin.

32. **Influence of Self Light on Perceptivity for Intensity Difference. Sensitivity.**—Helmholtz investigated the effect of the irregularly-flecked luminosity of the retina (this must not be mistaken for actual luminosity in the retina; it refers to a brain impression originated not by present external light,

but by some action occurring in the retina or the brain; which action may be due to precedent illumination) upon the sensitiveness of the eye for the detection of differences in the intensities of external sources of light.

If I be the intensity of the external stimulus which would be necessary to produce the same sensation as that given by the intrinsic luminosity of the retina at any point, while ds is the area of the retina upon which the stimulation lies within the limits I and $I+dI$, the magnitude of ds is proportional to dI, and is otherwise a function of I alone, say $f(I)$. The whole area under consideration is therefore

$$\int_0^{I_1} f(I)dI = A,$$

where I_1 is the greatest value that I has at any point of the retina; and the mean value of I over the whole of this area is given by

$$AI_0 = \int_0^{I_1} If(I)dI.$$

But, for an actual external stimulation di, Fechner's law gives

$$dS = di \int_0^{I_1} \frac{f(I)dI}{I_0 + i}.$$

If I_0 could be regarded as constant for any value of i, this would give

$$dS = A\frac{di}{I_0 + i},$$

which is Fechner's form of the relation. But, if I_0 depends on the external stimulation i, we may write instead of it $I'+i+a$, where I' represents the mean value of I_0 when account is taken of all the values of i over the illuminated area of the retina, and a is the deviation from the mean. Expanding in a series of powers of a and neglecting terms involving the fourth and higher powers of the ratio $a/(I'+i)$ we have

$$dS = di \int_0^{I_1} \frac{1}{I'+i}\left(1 + \frac{a^2}{(I'+i)^2}\right)f(I)dI.$$

The terms in odd powers of a do not appear because their

mean value in the integration is zero. Writing A′ as the mean value of a^2 over the whole area we get

$$\frac{ds}{di}=\frac{1}{\mathrm{I}'+i}\left(\mathrm{A}+\frac{\mathrm{A}'}{(\mathrm{I}+i)^2}\right).$$

Hence

$$y=\frac{di}{ds}=\frac{(\mathrm{I}'+i)^3}{\mathrm{A}(\mathrm{I}'+i)^2+\mathrm{A}'}=\frac{x^3}{\mathrm{A}x^2+\mathrm{A}'}.$$

The error introduced by neglect of the higher terms would not exceed 1 per cent. even if I_0 differed from I′ by about 32 per cent. With suitably chosen constants the curve obtained by plotting y against x agrees practically, for positive values of i, with that given by Helmholtz as the result of a somewhat different approximation. It exhibits satisfactorily, as Helmholtz remarks, general features of the phenomena, though it is too flat at the bend of the curve.

As the intensity of the light becomes large, the eye becomes dazzled and the deviation (shown in the upward trend of the curve at high intensities in Fig. 1) from Fechner's law becomes marked. Helmholtz showed that a small linear correction term gives a good account of the observed deviation until the dazzling becomes so great that observations are largely valueless.

The *sensitivity* may be defined as the ratio of the intensity to the change of intensity per unit change of sensation. It is thus measured by the ratio of i to y. But, at large intensities, the term involving A′ is negligible, and so the sensitivity, σ, is

$$\sigma=\frac{\mathrm{A}}{\mathrm{I}'+i}.$$

Dazzling makes A vary slightly instead of being constant, the varying value being well represented by $\mathrm{A}/(1+ki)$. Thus we get Helmholtz's extension of Fechner's law in the form

$$\sigma=\frac{\mathrm{A}}{(\mathrm{I}'+i)(1+ki)},$$

where k is a small constant.

33. Dependence of Self Light on External Stimulation.—The apparent luminosity which is evident to the eye against

the dark background of a room from which light is totally excluded depends upon the illumination which has previously fallen from external sources upon the retina. An effect of this kind might arise from fluorescence in some part of the eye, or from the persistence of that physical consequence of the illumination of the retinal structures which gives rise to the stimulation of the nerves, or from persistence of that stimulation, or from persistence of the physiological effects of that stimulation when it influences the brain organs concerned in psychological perception. On the other hand, it may have its origin in the retina itself from an extraneous development therein of the physical action which results, to a still greater extent, when external light falls upon the eye : or it may originate by extraneous development of any of the energy transformations which arise in the transmitting apparatus or in the brain itself as a consequence of the incidence of external light upon the retina. If that extraneous development be to any extent independent of precedent external stimulation, self light of the eye should be evident even after the most prolonged rest. If it be not thus independent, the rested eye should ultimately acquire full perceptivity for weak illumination in accordance, for example, with the unmodified expression of Fechner's law (§ 109).

Temporary persistence of the effects of external stimulation is very evident in the case of after images (Chapter XIII), and in such phenomena as the luminous glare which prevents the perception of objects for some time after one re-enters a well-lighted house subsequently to exposure of the eyes to strong outside daylight. Consequently it cannot be expected that the quantity I' in the expression for y, can be constant. In other words, the expression found for y can only approximately (quite apart from omission of terms in the expansion) represent the experimentally found results.

The representation should therefore be capable of improvement by choice of a suitable expression for I' as dependent on i, while retaining the general form given to y as dependent on i, although this procedure is not strictly correct. Further, the known way in which the "glare" above referred to increases under external stimulation, and the known phenomena of "fatigue" (§ 87), would lead us to expect that the term I'

might at first increase with increasing stimulation at a rapid rate and thereafter decrease slowly to a constant value.

The initially lower curve in Fig. 2 represents the value of y

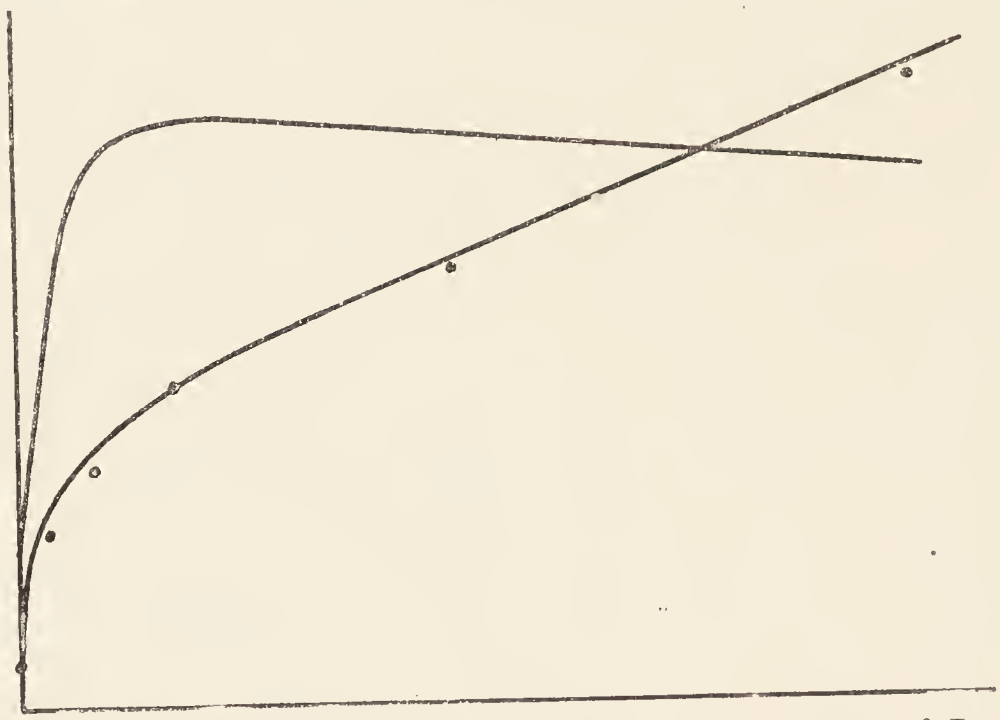

FIG. 2.—Effect of Self Light and Fatigue on the Laws of Intensity.

if instead of a constant value for I′ we choose the purely empirical expression

$$I' = 5\left[1 + 1{\cdot}36\frac{i - \frac{1}{2}}{(i+1)^2}\right] + i.$$

Very close correspondence with observation is apparent.

The other curve represents

$$I' = 5\left[1 + 1{\cdot}36\frac{i - \frac{1}{2}}{(i+1)^2}\right]$$

and shows that a purely empirical expression for the effect of self light, chosen so as to give close agreement between the results of calculation and experiment, exhibits that characteristic trend with increasing i which has just been indicated as probable in consequence of increase of retinal changes to a maximum together with slow increase of fatigue.

In subsequent chapters the importance of the modifications of visual perception by the presence of self light will be found to be prominent.

34. **The Sensation of Colour : A System of Three Freedoms.**—The simplest and most fundamental fact connected

with colours is the fact, already fully discussed, that any colour, if suitably darkened by the addition of black or diluted by the addition of white, can be matched by a mixture of three suitably chosen fundamental colours. Simple results imply simple precedents, and no view wherein recognition of this essential feature of colour vision is not made can form an adequate treatment. Discussion has also been made (§ 27) of modes of representing the results of colour mixture. A point P, which is displaced outwards from a point O taken as origin, has its position completely specified by giving its component displacements along three mutually perpendicular axes passing through O. We may take the components as, say, x units of length eastwards, y upwards, and z southwards. Or, more explicitly, the displacement of magnitude d in the direction D is

$$d\mathrm{D}=x\mathrm{E}+y\mathrm{U}+z\mathrm{S},$$

the letters D, E, U and S being mere symbols representing direction. Now we may use this framework of lines to represent any quantity which has three independent components, say Red, Green and Blue. Thus we may assert that the amount c of a colour whose quality is C is compounded according to the condition

$$c\mathrm{C}=x\mathrm{R}+y\mathrm{G}+z\mathrm{B}.$$

The number of units of R is representable by the number of units of the corresponding component in the geometrical framework. If we lay down the condition that the magnitude of the intensity of the coloured light is to be unity we shall have $x+y+z=1$, and the point which represents the resultant colour will lie somewhere in the plane which passes through the three points $x=1$, $y=0$, $z=0$; $x=0$, $y=1$, $z=0$; $x=0$, $y=0$, $z=1$. If the magnitude be greater than unity, the point will lie on a plane parallel to this one but more remote from the origin; and conversely if it be less than unity. In particular the origin represents a stimulus of zero magnitude.

If we intersect the framework by such a plane, equally inclined to all the axes, and stand it on that section placed horizontal, the framework outlines a pyramid—*Lambert's Colour Pyramid*. Any point within it represents a colour compounded, in accordance with Newton's law of colour mixture, of definite amounts of the three fundamentals. All stimuli being regarded

as positive essentially (as they are on the Young-Helmholtz theory) which is the simplest presumption and is found to be sufficient, no point outside the pyramid, in the direction of any or all of x, y, z negative, can represent a real colour.

If the colour be independent of the intensity, all points on any one straight line passing through the origin represent the same colour. This is practically the case throughout a very considerable range of intensity, though it does not hold under very weak or very strong illumination. Within that range, therefore, any particular colour is completely specified as to its proportionate contents of R, G, B by the single point in which the line of that colour drawn from the apex of the pyramid, cuts the base. Instead of using the pyramid in these cases, therefore, we may use the triangular equilateral base, and so obtain a geometrical representation of the compounding of colours by a plane diagram, which is more convenient. This is *Maxwell's Colour Diagram* or *Colour Triangle.*

If unit quantities of R, G, B be supposed placed at the corners, the perpendiculars drawn from any internal point to the three sides give the *proportions* of the three fundamentals present. Their *amounts* can be determined by the condition that the total magnitude is unity. This condition is realized by taking the triangle of such size that the length of a perpendicular from an apex to the opposite side is unity. The sums of the lengths of the perpendiculars drawn from any internal point to the sides is then unity.

35. **The Position of White.**—In the measurement of three component *lengths* along three rectangular directions no purpose could in general be served by using different units of length for the three different measurements. But, when we measure the amounts of three independent coloured lights along three rectangular axes, or perpendicular to the sides of an equilateral triangle, it may be convenient to use three different units. If, for example, in the stimulation of the eye by white light, the three fundamental sensations were not equally affected, it might be convenient to employ different units for each so as to make the numerical magnitudes equal. We would then have the simple expression for unit intensity of white

$$\bar{W} = \tfrac{1}{3}R + \tfrac{1}{3}G + \tfrac{1}{3}B.$$

Similarly any colour C, which is specified by

$$C = xR + yG + zB$$

can, if, e.g., z be the smallest of the three magnitudes, be expressed as

$$C = (x - z)R + (y - z)G + z\overline{W}.$$

Thus any colour can be expressed in terms of two fundamentals together with White. This point is of importance in connection with some theoretical views (§§ 38, 44).

The introduction of the convention has the diagrammatic advantage of placing the unique white colour at the centre of the colour triangle.

The units in which the three component stimuli are measured being entirely arbitrary, and independently so, we can choose three units to suit any desired purpose: we can even choose one set for one purpose and another set for a different object. But we have no means of directly measuring a sensation: it can only be expressed in terms of its effective stimulus. Yet we can, by suitable choice of the units for the component stimuli, introduce one element of concord between the measurement of stimuli and the expression by these measurements of the ensuing sensations so as to express the condition that white light stimulates the three fundamental sensations equally. The question is entirely one of *convenience* of definition.

36. **The Positions of the Spectrum Colours.**—If the positions of the corners of the colour triangle are fixed, the spectrum colours are also fixed. Since it is postulated that all fundamental sensations are necessarily positive, no part of the spectrum can lie outside the triangle. If three colours of the spectrum were taken as the three fundamentals, the remaining colours must either lie on the sides of the triangle or on a curve exterior to it, but passing through the corners. If any portion of the spectrum curve be rectilinear, the linearity of the law of colour mixture shows that the colours in that portion must be compoundable from the colours at its extremities. If, on the other hand, the portion of the curve lying between two spectrum colours be concave to the centre of the triangle, the intermediate spectrum colours cannot be matched by a mixture of the two. White must be added to them in order that a match may be reached. Conversely, if the inter-

mediate stretch were convex to the centre, a mixture of the two colours would require to have white added to it in order that it might match the spectrum colour which lies between its position and the centre.

In actual fact it is found that the colours obtained by mixture of any comparatively near spectrum colours are in general whiter, and are never less white, than the corresponding intermediate spectrum colours. Therefore the curve of the spectrum colours is in general concave to the centre of the triangle. The exceptional parts lie towards the extremities of the visible spectrum, and they are practically rectilinear.

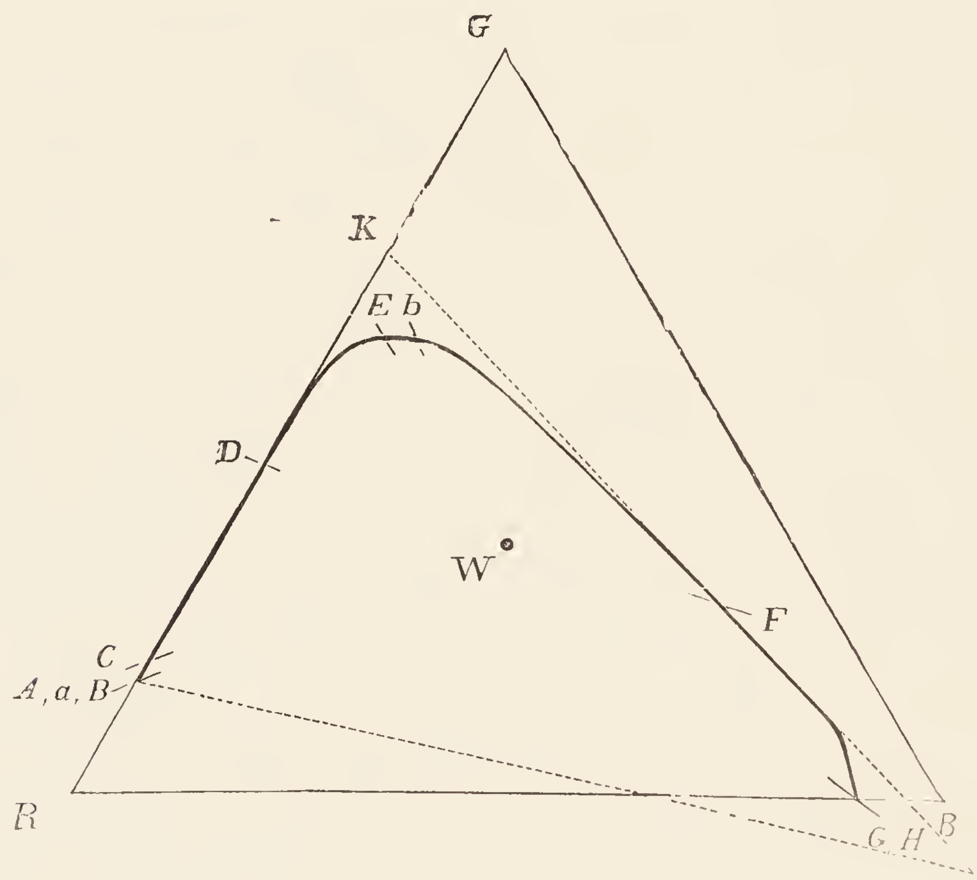

Fig. 3.—Spectrum Curve in the Colour Triangle.

The assumption which Newton made when he drew the spectrum curve as part of a circle (without reference to fundamentals), was that corresponding mixtures of any two equidistant colours in the spectrum were less pure, i.e., were whiter, than the corresponding intermediate spectrum colour, and that to exactly the same extent. In the absence of experimental knowledge that was the simplest condition that could be assumed.

If we choose the extreme red of the spectrum as one of the

fundamental colours, and the violet as another, the above considerations show that the third fundamental may lie at the intersection of the prolongations of the two long rectilinear portions of the spectrum curve. This is exhibited in Fig. 3. The sharp inflection of the extreme tip of the violet end of the curve is due, as Helmholtz pointed out, to fluorescence in the eye which makes the light falling on the retina from this region of the spectrum whiter than the actual light as it would be perceived by a non-fluorescent eye.

37. **Visible Colours.**—Only a certain range of wavelengths of light is visible to the normal eye, and every visible light is compounded of these in various proportions. Therefore it must not be supposed that every point in the interior of the colour triangle corresponds to an actually possible sensation. On the Young-Helmholtz theory all sensations are necessarily positive ; but all positive sensations are not necessarily experienced by a fully perceptive eye, nor are they actually experienced by it. The limitations are imposed by the mechanism of colour perception. The apparently arbitrary limitation of sensation to positive values, instead of being over-restrictive, is actually wider than is requisite.

The usually visible colours are those included within the boundary formed by the spectrum curve and the straight line joining its extremities. This line is the locus of the purples, ranging from red to violet, which form the only natural colours not included in the range of the spectrum. Each visible colour in nature or artificial representations is compoundable of some one colour on the boundary along with white. For every point within the boundary can be so represented in accordance with the Newtonian law of colour mixture.

38. **Choice of the Fundamentals.**—There is no a priori reason why the extreme colours of the visible spectrum should be chosen as true fundamentals. If spectrum green, necessarily as we have seen, contains white relatively to the green fundamental, it is a matter for experiment if possible to determine whether or not there is also white in spectrum red and spectrum violet. Now it is a fact that the phenomena of fatigue (§ 88), enable us to obtain in certain circumstances, from spectrum red or spectrum violet, a sensation of purer colour glow, i.e., a less white sensation than that normally produced by these

colours. And that is true also in the case of all other spectrum colours.

This means that the spectrum curve, above discussed, has a definite localization in the colour diagram under definite conditions only. Under normal conditions of vision it has a definite position. But the condition of visibility can, under special circumstances, be extended outwards from the centre of the triangle. The above-mentioned boundary of visible colours is therefore only the normal boundary, and it must be included entirely within the true triangle of the fundamental perceptions.

This fact, that white is present in all visible colours, is in part the cause of the supposition, not unfrequently made, that white is itself one of the fundamentally independent perceptions. The Young-Helmholtz theory gives a direct and simple account of it without any such added complication.

The experimental condition that the three fundamentals really lie outside the range of normal vision makes it, so far, an entirely secondary question at what points they really lie. That, on the average over a sufficiently great number of eyes possessing "normal" vision, they will be found to lie at practically definite points is nearly certain. But such facts as those alluded to in § 14, regarding so-called normal vision, makes it a certainty that these points must differ greatly amongst individual eyes.

The indefiniteness of the fundamentals, as amongst individual eyes, receives a very ready explanation on the Young-Helmholtz theory. If the linear law of colour mixture holds, as it does to at least a very high degree of approximation, in the case of each individual eye, any one colour is expressible, for each eye, linearly in terms of the fundamentals of that particular eye. Therefore the fundamentals of one eye can be expressed as linear compounds of the fundamentals of another. And it is a matter of no primary consequence which set is employed. Diagrammatically this means that any colour triangle which includes within it the boundary of normal vision can equally well serve descriptive purposes. The three corners may be always fixed in the case of any one eye; but, on the other hand, they may possibly vary from one time of

life to another, or even from one day to another. Such variations might be due to differences of sensitive materials in the retina, or to cross "leakage" of effects in the transmitting mechanism, or to cross leakage in the cerebral mechanism.

The question of indirect determination of the fundamentals in the case of a given eye is dealt with in Chapter IX.

CHAPTER V

TYPES OF TRICHROMATIC VISION

39. **The Absolute Colour Triangle.**—In spite of the indeterminateness of the problem of fixing the effective fundamentals for any particular eye, it may be the case that there exist three absolute fundamentals each corresponding to the hypothetical case of excitation of one alone of the three brain centres. If there were no variations in the characteristic qualities of these three centres amongst human beings, these hypothetical solitary stimulations would be identical for all. It is of interest to inquire whether or not there may be observational or experimental evidence which favours the supposition. To settle this point it is necessary to inquire into the consequences of the postulate.

40. **Characteristics of Normal Trichromasy.**—Under normal conditions the law of colour mixture shows that a colour triangle which is descriptive of the individual colour perception, exists. The whole of its internal area need not, and in general does not, correspond to actual perception. Under definite conditions the area of perception is bounded by the spectrum curve and the line of purples. But the boundary varies under different conditions of stimulation, though it is very constant under a wide range of conditions. When the intensity of external stimulation is excessively weak, the boundary tends to shrink in to the centre ; and, when the stimulation is very intense, it also tends to shrink in to a point. As we have just seen in the last chapter, it can be expanded outside its normal location under special conditions preceding stimulation.

Since the middle ranges of the sides of the colour triangle necessarily approximate more nearly to the central white than do the corner stretches, it may be possible that this condition corresponds to a visual peculiarity which appears also in the

visual boundary, and so in the spectrum curve. A peculiarity of this kind is found in the spectrum curve, and agrees with the law of colour mixture in justifying the postulate of normal trichromasy. Three regions of greater purity separated by two of less purity, i.e., of closer approximation to white, exist in the normal spectrum. But the fact that a considerable stretch of wavelengths at either end of the spectrum produce unchanged colour impressions (Fig. 3), so that these stretches are represented by single points in the colour diagram, does not necessitate the supposition that these points correspond to absolute, or even to individual, fundamentals. The nature of the interaction between light and the light-perceptive mechanism is involved. Indeed, in consequence of this, it is utterly impossible to assert that the purer spectral regions must necessarily lie in the same direction, as seen from the centre, as do the fundamentals. Such considerations show how entirely futile it is to suppose that we can judge of the closeness of spectrum colours, or non-spectrum colours such as the purples, to fundamentality by our feelings regarding the aspect of these. Even if the individual fundamentals did correspond in respect of orientation in the diagram, to the three purer regions of the spectrum, the absolute fundamentals, did such exist, might be exactly inverted and correspond to the less pure parts—yellow, middle blue, and a purple.

41. **Variation of Trichromasy.**—Consider the case of an absolute triangle, and the formation from it, by interfusion of effects, of an individual triangle of colour perception.

The condition of no negative stimulation of the absolute fundamentals is satisfied if the three derived fundamentals lie inside the absolute triangle. The position of any point in the derived triangle corresponding to a given point in the fundamental triangle can be readily found. To any line passing through a corner of the one triangle, and intersecting the opposite side so that its segments are in a given ratio, corresponds the exactly similar line in the other triangle. The derived triangle will not in general be equilateral ; but a representative equilateral triangle can readily be constructed from it, to exhibit the usual individual triangle, though it could not then show the relation to the absolute quantities unless the absolute triangle, now distorted, were drawn in also (Fig. 4).

This single condition of the derived triangle being internal to the other can include all cases of ordinary trichromatic vision, but it also includes other cases of very abnormal trichromasy. For not merely may the triangle be variously deformed from equiangularity : it may also have the order of occurrence of the corners inverted. For example, there might be complete exchange of red and green causative substances in the respective nerve endings. In this case the red end of the spectrum would be seen as green, and the green as red. And three such types are possible.

Except as a result of subsidiary effects, such as different crowding of wavelengths in different parts of the spectrum, vision of this type would seem to be fully trichromatic. For, though the red part of the spectrum would be seen as green, the orange as yellow-green, the yellow as yellow on the whole, the green as red, the blue as purple, and the violet as violet, the nomenclature to which the individual had been trained would prevent confusion of description as between this abnormal eye and the normal one. Even in a case in which the normal, and this abnormal, condition subsisted respectively in the two eyes of one individual, though the fact should be readily detectable, it would be impossible to tell which eye was normal and which abnormal apart from subsidiary effects. The binocular vision would in this case be that of the most usual type of colour blindness.

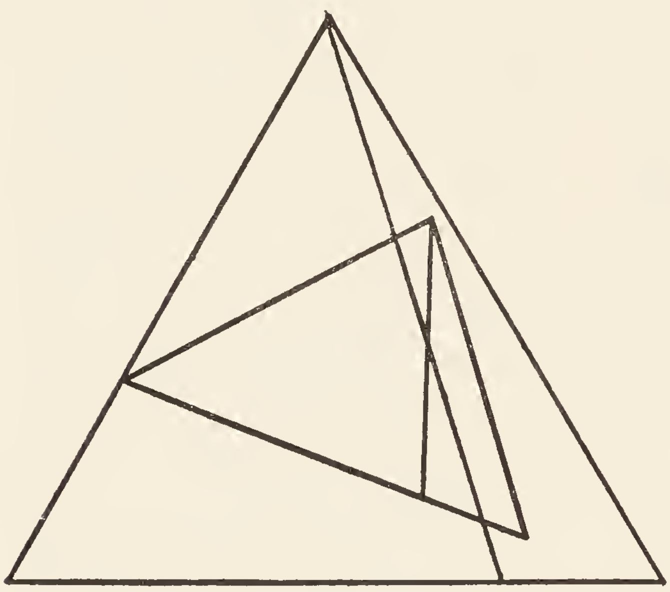

Fig. 4.—Derived Colour Triangle. Class I.

42. **Trichromasy and Quasi-Multichromasy.**—The restriction that the corners of the derived triangle shall lie within the absolute triangle, or at least not external to it, is quite arbitrary. Considerable tracts of the area within the derived triangle lie outside the region of visual perception. All that is necessary in the derivation of an individual triangle is that no part of its area of perception shall lie outside the

absolute triangle if the visual mechanism is such that no negative absolute stimulation can occur and the cerebral mechanism is normal. The question of the nature of the structural or functional conditions, which may lead to an external position of a corner of a derived triangle relatively to the absolute triangle, lies altogether apart.

The possible cases occur under four classes. The first, already discussed, is that with the three derived corners inside the absolute triangle ; in the second two corners are inside ; in the third, one is inside ; in the fourth, all three are outside. In these classes various groups appear ; for the prolongations of the sides of the absolute triangle divide the area external

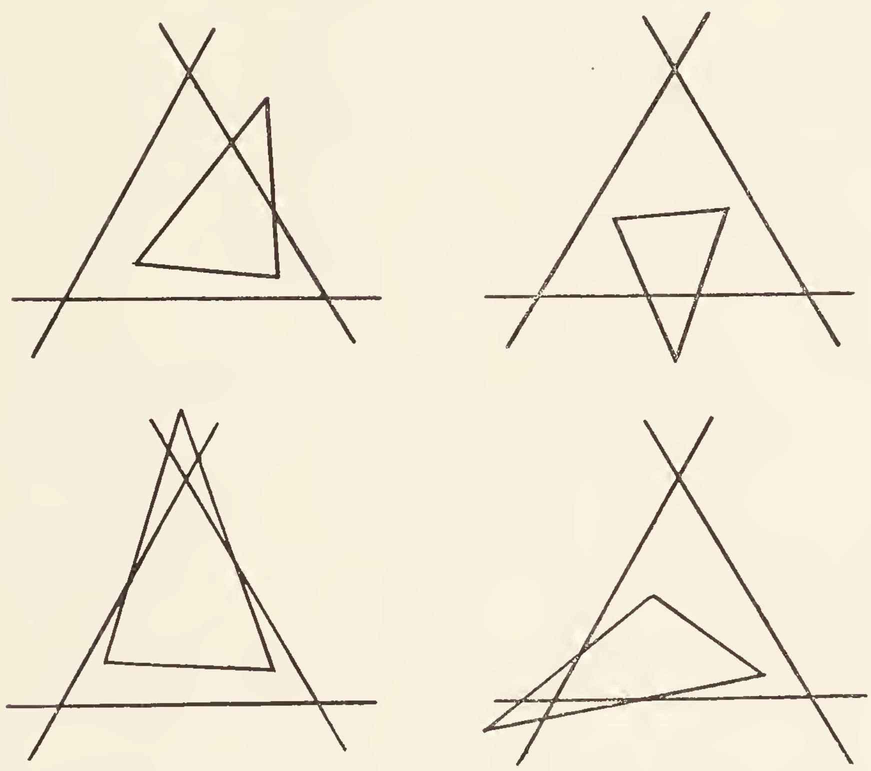

Fig. 5.—Derived Colour Triangles. Class II.

to the triangle into two sets of three regions, and the corners of the new triangle may be variously positioned therein. Also the order in which the fundamental perceptions occur relatively to the spectrum colours may, as we have seen, be inverted.

Fig. 5 represents some of the possibilities in the second class. The areas common to the two triangles represent the regions in which the absolute and the derived stimulations are alike positive. We may call this region the Colour Polygon. In this class it can only be a quadrilateral or a pentagon.

Some cases belonging to the third class are shown in Fig. 6. Here the polygon may be a triangle, a quadrilateral, or a

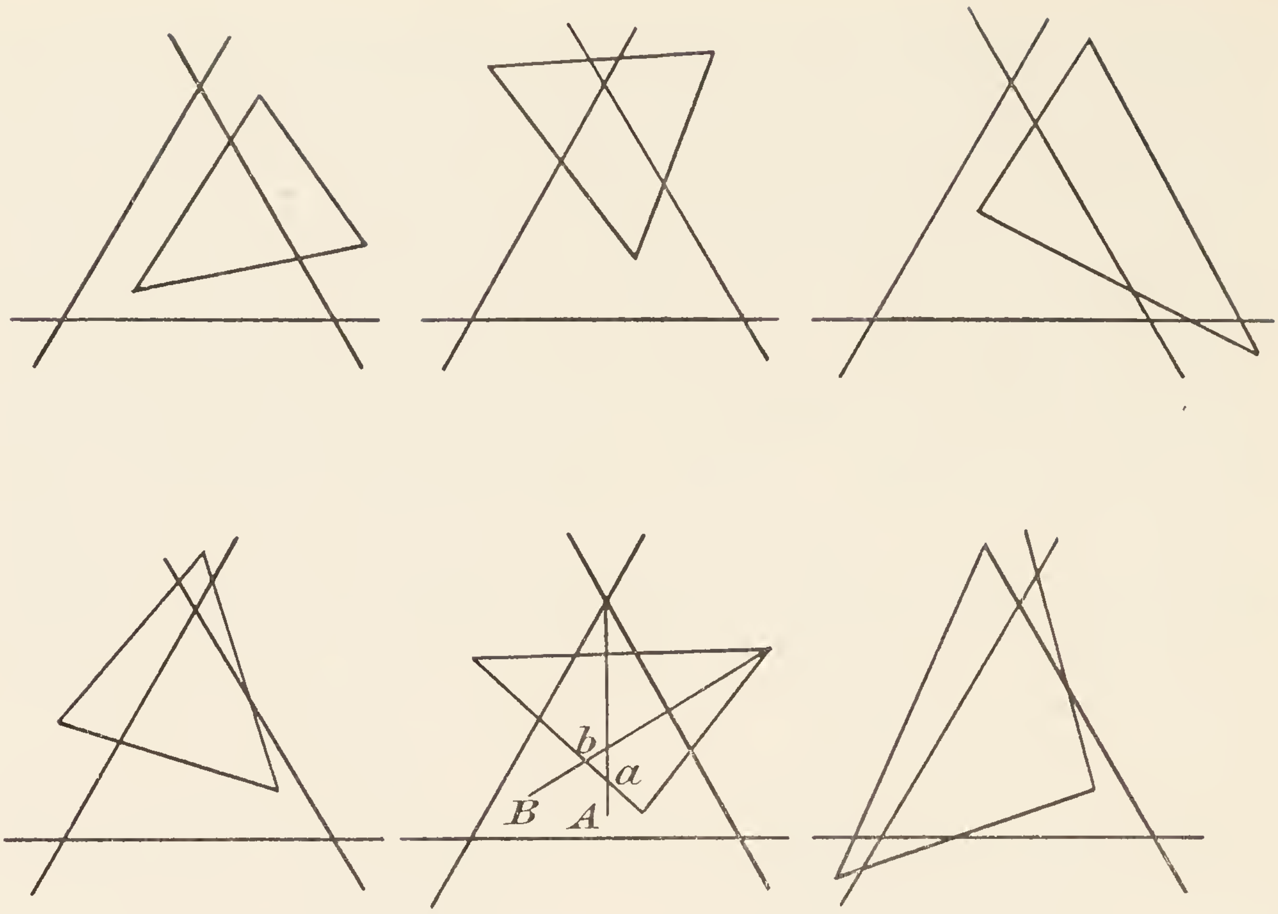

FIG. 6.—Derived Colour Triangles. Class III.

pentagon. The fourth class contains cases such as those in Fig. 7. In this class the polygon may be a triangle, a quadri-

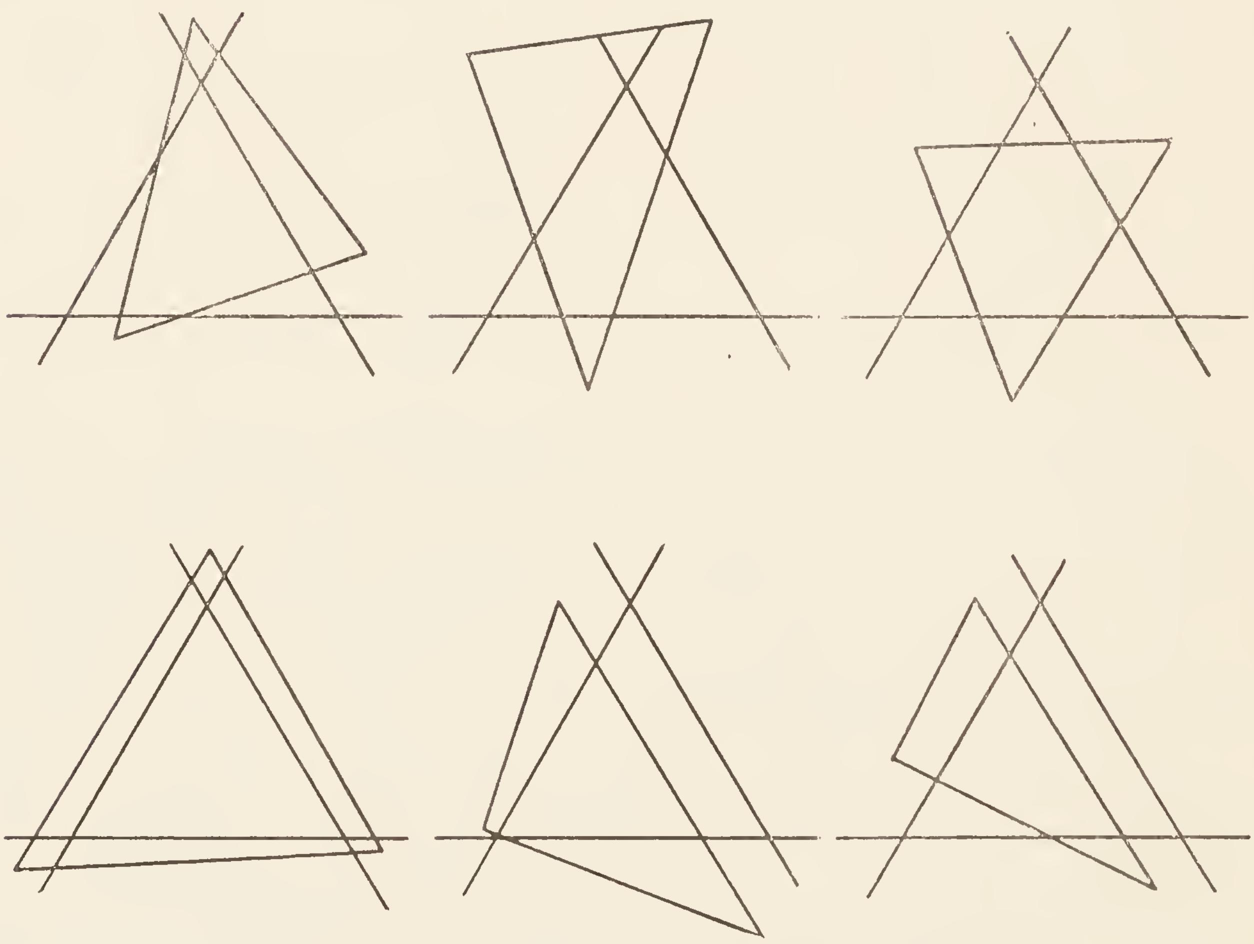

FIG. 7.—Derived Colour Triangles. Class IV.

lateral, a pentagon, or a hexagon. The latter can occur in one way only apart from inversion of order in spectrum colours. But its varieties range from the distinctly hexagonal to the approximately pentagonal, quadrilateral, or triangular. The diagrams are merely illustrative, for the possibilities from all the classes are great.

We have now to consider what actual or seeming characteristics of vision may be produced in these cases.

Apart from the possible inversion of spectrum colours there may be telescoping of areas of the visual region into lines. Any colour

$$c\mathrm{C}=x\mathrm{R}+y\mathrm{G}+z\mathrm{B}$$

represented by a point outside the R, G, B triangle where, say, z is negative cannot, according to the fundamental assumption, produce any negative B action, so that z has to be put equal to zero, and the colour is actually

$$c\mathrm{C}=x\mathrm{R}+y\mathrm{G}.$$

Thus, in the preceding figures, any colour which otherwise would have its representative point in a region of one triangle which is located beyond a side of the other triangle, has its actually representative point at the intersection of that side with the straight line joining the point to the opposite apex. For this construction gives the correct ratio of the two stimulations which, being positive, actually exist. Thus, in Fig. 6, the points a and b represent the actual stimulations which correspond to the points A and B, the points which would be representative were negative stimulations possible with regard to the derived or the absolute fundamentals. Negative stimuli are inoperative. Thus we see how interfusion of the absolute fundamentals may give rise to inhibition (§§ 94, 97) of perception with regard to tracts of colour.

The outstanding features of these consequences of interfusion or interaction of fundamental stimulations, to whatever cause, structural or functional, these may be due, is that the individual visual boundary may present relations towards the sides and apices of a polygon of any order up to the sixth inclusive, instead of towards those of a triangle only: and, in these cases as in the latter, the vision is nevertheless purely trichromatic.

The polygonal boundary need never be referred to. Any sufficiently inclusive triangle would suit.

This multiplicity of apices and sides may be a constitutive, but, on the trichromatic theory, unreal basis for the view that colour vision is multichromatic. But even with regard to a triangular representation there is room for a pentachromatic description of the spectrum related to the three apices with their more saturated colour impressions, and the mid points of the sides with their less saturated impression. More and less saturation here really refers only to greater or less remoteness from the centre of the triangle, which is assumed to be capable of psychical discrimination.

43. **The Colours of the Spectrum.**—We may conveniently speak of spectrum colours as simple or fused quite apart from any question of their impurity or admixture with white. In this case a simple colour is one which cannot be compounded from the others. Red and indigo towards the extreme ends stand out characteristically as such. The other is green, since it can only be compounded of two spectrum colours each of which perceptibly contains green together with a constituent which is complementary to the corresponding constituent of the other. Admixture of green and indigo gives blue, and addition of a little red to indigo gives violet. So also addition of red to green gives yellow: and these two fused colours, yellow and blue, contain, to the normal eye, no trace of a constituent colour. They are less pure, in the sense that they contain more white than the component colours from which they are derived; but, so far as eye judgment is concerned, they are as distinctive in colour as are the simple uncompoundable colours from which they are derived. The other fused colours are the purples obtainable from the addition of red to indigo or violet or blue.

But we might take the fused colours yellow, blue, and purple as simple colours, and fusion of these in pairs would give the former simple colours. Thus yellow fused with purple would give red, the yellow being complementary to the blue or indigo in the purple; and so on. This is a direct result of the linear law of composition of three fundamental colours. It is not possible, from phenomena of colour mixture, to determine the fundamentals. The only reason for choosing red, green, and

indigo or violet, from amongst spectrum colours, as simple and fundamental colours (corresponding, that is, to fundamental sensations) is that they have the smallest admixture of white: but we have no a priori reason for asserting that the parts of the spectrum curve most remote from white must lie towards the apices of the absolute colour triangle (§ 40).

On the other hand, the characteristics of spectrum colours as seen by any eye may exhibit the relation of the arbitrarily chosen fundamentals to another set from which they are derived. And if sets of three fundamentals, synthesised in this way from observations on spectrum perception by many variously percipient eyes, were all in close agreement with each other, the probability would be strong that the absolute fundamentals were indicated.

The positions of the Fraunhofer lines on the spectrum curve are indicated in Fig. 3 (p. 54) by lines intersecting the curve. All wavelengths beyond A towards the red end of the spectrum are superposed at the corner A of the closely circumscribing triangle AKV: all beyond G towards the violet end are superposed at that point. The three points A, K, V indicate the positions of the arbitrarily chosen fundamentals. No reason exists for such a choice apart from the idea that the curve of the natural sunlight spectrum possibly might be expected to deviate little from the sides of the natural colour triangle. But the other extreme supposition that the deviation from white should be minimal indicates a relationship which is as simple from the point of view of a priori postulation.

The equilateral triangle shows the fundamentals determined by König and Dieterici so as to satisfy the condition that typical cases of red and green "blindness," so called, should be describable as due to failure of one of the normal fundamentals.

It would be possible to use alternatively a colour hexagon, drawing through A a side perpendicular to AD, through G a side perpendicular to the inflected rectilinear part of the spectrum curve, taking the three rectilinear parts of the curve, and finally a straight line, very arbitrary in position, cutting off the corner K. The sides of the hexagon coincide with the sides of A, K, V regarded as a derived triangle, together with the sides of a presumed absolute triangle. The fact that the

existence of the one side is ascribable to the action of fluorescence would alone introduce doubt: and the known fact that it is possible, through the action of contrast, to have vision corresponding to points more remote from white than the normal spectrum curve at least compels widening of the hexagon.

There is evidently unavoidable inconclusiveness in any attempt to determine the absolute fundamentals without the employment of a process which depends on a non-linear effect which is the logical outcome of the experimentally established

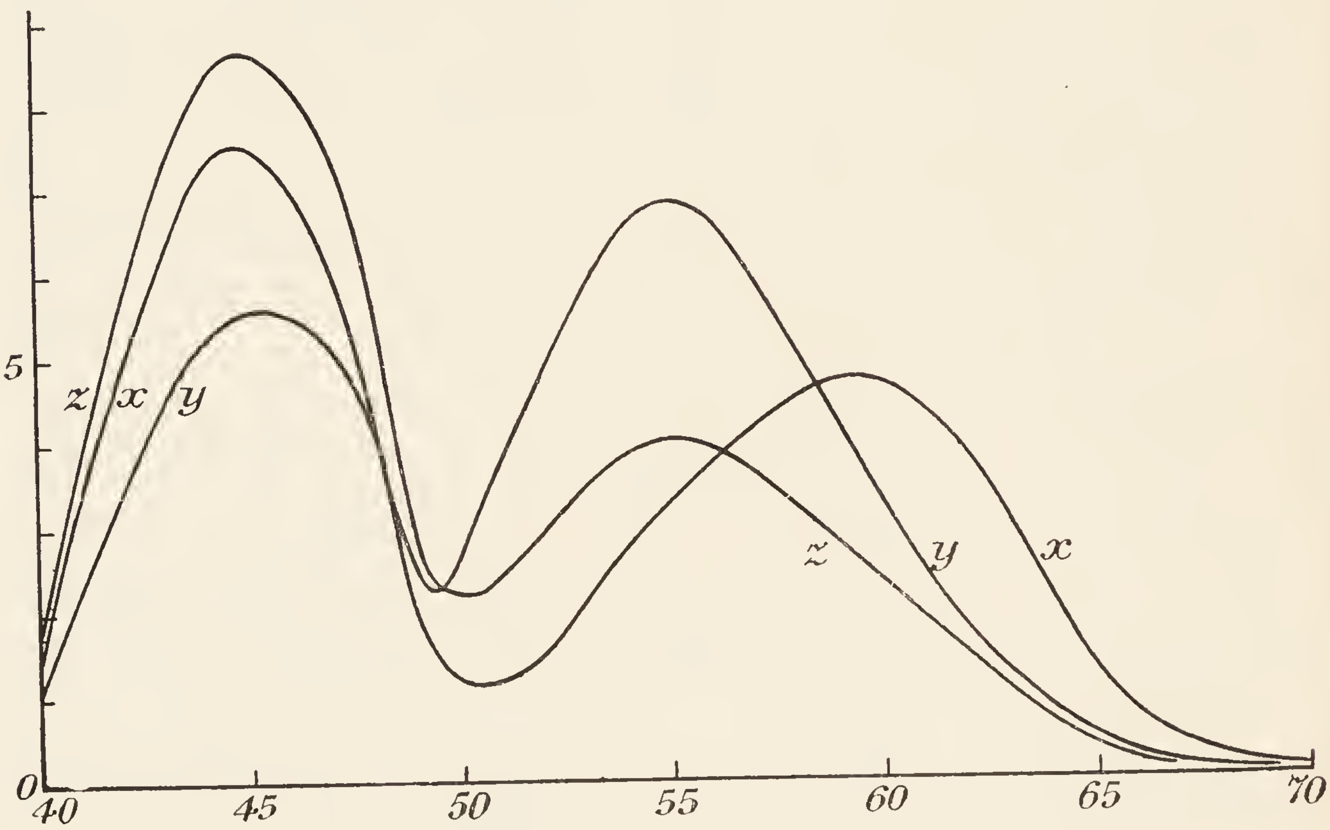

Fig. 8.—Spectrum Stimuli in Helmholtz's Absolute Triangle. The curves x, y z represent respectively the red, green and blue stimuli.

laws of vision. This process was adopted by Helmholtz whenever observational results made the necessity for it obvious, and at the same time made its adoption possible. The ratios of the ordinates in Fig. 8 are the ratios of the perpendiculars on the sides of his absolute triangle.

All that would in general be predicable is the direction, as seen from the centre of inertia of the spectrum curve, of the points of maximum and minimum purity, i.e. of maximum and minimum distance from the centre of inertia.

Peculiarities of vision corresponding to the various diagrams of Figs. 5, 6, and 7 can readily be deduced if the position of

the spectrum curve be given. The characteristics of the vision may be found by consideration of the outstanding colour characteristics of ordinary vision. Rich pure colour glow is the characteristic of the simple colours, say, red, green, and indigo; strong but whiter colour distinguishes the fused colours yellow and pure blue. These normally seem to be quite distinct colours from the colours of which each is compounded in equal parts. On the other hand, the colours intermediate in the spectrum between a simple colour and a fused colour partake of the qualities of both. Thus orange can be described as a yellow red or a red yellow; between yellow and green we have the green yellows and the yellow greens; and between green and blue we have the blue greens and the green blues.

44. **Special Cases of Trichromatic Spectra.**—First consider the case in which the colours R and B, taken as red and blue, the latter deeper in tone than the compound blue above referred to, are unaltered, while green, G, becomes the yellow complementary to the blue B. In this case the lower half of the absolute triangle becomes the colour triangle. Its centre of inertia is displaced from the normal centre towards the red

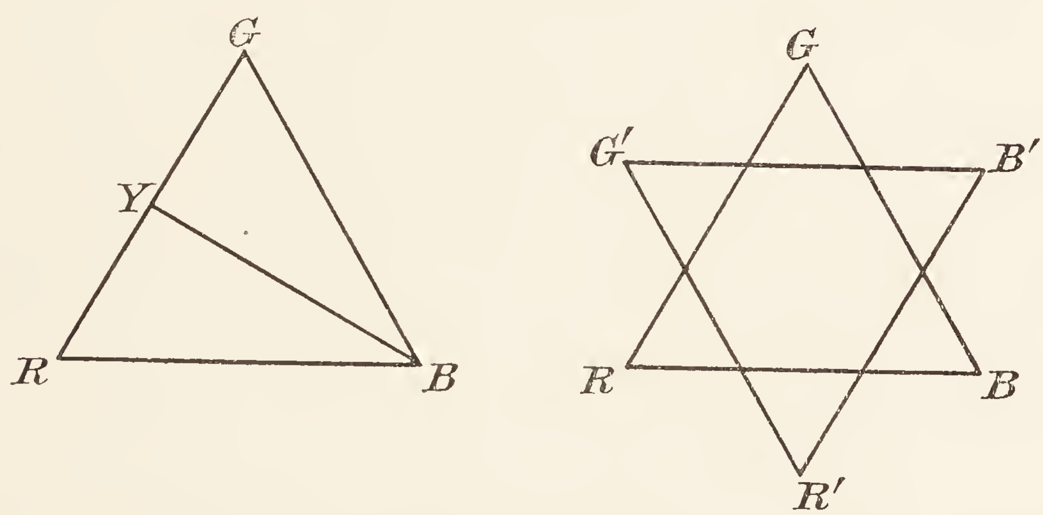

Fig. 9.—Special Derived Colour Triangles.

region. Thus the stimulation which produces the impression called white in consequence of training, and which is seen as white by the normal eye, is a stimulation which produces a reddish white impression on the normal eye. Also, while two of the new fundamentals are still "simple" colours, the third has become a "fused" colour—yellow, which is nevertheless

by training called green, and is stimulated by the wavelengths which normally stimulate green. So the identical stimulations which normally extend over the spectrum range red to yellow now extend from red to green ; and the double colour characteristics of the intermediate yellow reds or red yellows now extend from red to green. The compulsory change in nomenclature is therefore found by replacing " yellow " by " green." Thus arise the " red-greens " of some trichromatic eyes. If B were a light spectrum blue, that is, a mixed colour, there would similarly be difficulty in colour terminology relative to the more refrangible half of the spectrum. It would probably be called " green-violet " with an intermediate colourless band. The band would be absent if B were purple, another mixed colour.

If green had been unaltered while R became Y, the upper half of the triangle, that is, the triangle YGB, would become the colour triangle ; and the same terminology with regard to red green would be used, for red is now actually yellow. In this case a red purple, whose representative point is at one-third of the distance along RB from R, produces on the abnormal eye the impression of normal white : but it is necessarily called a reddish purple. In fact all colours normally lying on the line joining the centre of the triangle RGB to that point on RB produce the same impression on this abnormal eye as white does on the normal eye. The term " green-violet," however, would not be used in this case if B were really a simple colour ; and, if it is lighter blue, a fused colour, the training in terminology would lead to the employment of the term " green-violet " in this case also.

Consider now the case in which, say, red becomes yellow, green becomes light blue, and fundamental blue becomes violet or purple ; so that the abnormal triangle is formed by the mid points of the normal triangle. Reds, greens and blues would seem to have strong colour glow and would be called red, green, and blue, though they produced a different impression from that made in normal vision. The intermediate colours would seem to vary from one to the other through a distinct colour in each case ; and these distinct colours would be called yellow, green-blue and purple respectively, while they actually are normal green, fundamental blue, and red respec-

tively. White would be quite normal, and the whole spectrum would be less pure than the normal spectrum. In consequence colour discrimination would be lessened.

It is instructive to consider further the case of green unaltered, while red becomes yellow and fundamental blue becomes light blue. Here the derived fundamentals consist of two fused colours and one simple colour. White would have a green tinge, and a purplish white would give to the abnormal eye the impression that white gives to the normal eye. The three colours red, green, and deep blue would be discriminated correctly through education in nomenclature, but the actual impressions would be those of yellow, green and light blue, the compound impressions of normal orange to greenish yellow and yellow green might be called red-greens ; and the compound impressions of normal bluish green to greenish blue might be called green-violets. Colour discrimination in these regions would be feeble especially if G were also altered so as to be whiter than normal.

Next, consider the derived triangle R′G′B′ in the second diagram of Fig. 9. The colour polygon is now a regular hexagon. At the mid points of its sides lie six colours in two groups of three, a member of each group being taken alternately. The one group consists of the three fused colours, yellow, green-blue, and purple ; the other consists of three derived from the simple colours, red, green, and blue, by addition of white. At the corners of the hexagon lie the intermediate colours, each distinguishable in terms of its two adjacent colours ; these are red-yellow or orange, yellow-green, bluish green, azure, violet, crimson. Of these apical colours some have received distinctive names, and the others might equally well receive them. The question of whether any one member of a contiguous pair of spectrum colours shall be more or less pure than the other is entirely a question of the departure of the various points of the spectrum from the central white.

In this case lie possibilities of a nonachromic vision more delicately perfect than the heptachromic vision of the Newtonian scheme. But even it, as well as the trichromic vision previously discussed, is really trichromatic in the Young-Helmholtz sense.

A typically interesting case arises from stimulation of all

three centres by action which normally should stimulate one alone, say the green. Here the three fundamental colours are red, white, and blue. Two normal "colours" alone are perceived, but the vision is not dichromatic. It is truly trichromatic, with white as an effective colour which the observer has been trained to call green. There would be little difficulty in discriminating by name the spectrum colours, but there would be great confusion of greens and purples which could both be matched with grey. A case of this kind has been described recently in the *British Journal of Ophthalmology*, by Dr. Edridge-Green.

It must be carefully borne in mind that abnormality of colour vision may theoretically arise in two distinct ways. A definite retinal action, which normally stimulates one brain centre alone, may stimulate others; or a definite centre, which normally is stimulated by one retinal type of action, may be stimulated by others. The effects are not necessarily the same. Thus complete admixture of retinal photochemical substances might give rise to colourless vision; but uniform partitioning of the action, due to one substance, amongst the three cerebral centres would not do so (§ 55). These effects are readily studied by means of the "sensation curves" (§ 69).

CHAPTER VI

THE LAW OF COLOUR SENSITIVITY

45. **Relation of Sensitivities to Colour and Brightness.**—In his work on sensitivity to brightness, Helmholtz dealt with constancy of colour. Then he pointed out that the power of the eye which makes possible the perception of gradations of luminosity, must also, in accordance with Young's theory, enable us to recognize the difference between two unlike compound colours, for it teaches us that different quantities of the fundamental colours exist in the two. If one colour contains more red and the other more blue, and if the fundamental red and blue are perceptions co-existing together, the difference of colour tone between the two colours is settled when we can recognize the difference of their colour contents.

He asks, Can we base the discrimination of the various colour tones on the gradations of intensity of the three ground colours in the colours under comparison? Does the same step in perceptivity occur as with luminosity, or a somewhat larger or smaller one? New complication only arises, he says, in so far as at least two, sometimes three, differences of perception of various types are simultaneously present and co-active. He adds that regarding the nature of this co-action of simultaneous perception differences of various types we can only seek now to find a probable hypothesis and test its validity by its results (§ 8).

If dS be the magnitude of the difference of resultant sensation, while dS_1, dS_2, dS_3 are the differences in the fundamental sensations, dS can only vanish if dS_1, dS_2, and dS_3 all vanish. For if one only of these did not vanish there would be a resultant difference in the two sensations, and no compensation could occur by another sensation qualitatively different. He then points out that the simplest form (§ 8) of function which has this property is one in which dS is necessarily

always a positive, homogeneous, and quadratic function of dS_1, dS_2, and dS_3.

Since small steps are considered terms of higher order do not require consideration, and he adds that if we, proceeding farther in accordance with Young's theory, postulate that the acts of consciousness only unite and mutually strengthen each other in so far as an objective difference of the same field is indicated by two or three impressions independent of each other and of different types, we shall have to omit in the quadratic function products of dS_1, dS_2, and dS_3. For their presence would indicate that the nature and sense of the physiological excitation had influence on the final result since two of the products would change sign if one of the quantities changed sign. The now upheld hypothesis aims, on the contrary, at expressing the condition that, in the mutual support of these impressions, it is only a question of the existence and magnitude of the influence upon the attention. We are thus limited to the assumption

$$(dS)^2=(dS_1)^2+(dS_2)^2+(dS_3)^2,$$

that is to say, the square of the resultant difference of sensation is the sum of the squares of the difference of each of the three component sensations. He points out that the reason for taking each of the co-efficients as unity is that dS must be equal to one component change if the other two component changes vanish.

He remarks that the formula shows that dS can step over the threshold value even if dS_1, dS_2 and dS_3 are somewhat below it, but that each component must exceed $1/\sqrt{3}$ of the threshold value. If the values of the three are very different, the smaller becomes ineffective, since the squares of small proper fractions are very small.

For most cases the unmodified form of Fechner's law (§ 28) would suit, but Helmholtz gives the generalization

$$dS_1=X\,\frac{dx}{x}\,\frac{1}{1+lx+my+nz},$$

$$dS_2=Y\,\frac{dy}{y}\,\frac{1}{1+lx+my+nz},$$

$$dS_3=Z\,\frac{dz}{z}\,\frac{1}{1+lx+my+nz},$$

which is required in special cases. These expressions differ from the unmodified form through the factors X, Y, Z and the common factor $1/(1+lx+my+nz)$. The quantities X, Y, Z are functions of the component stimuli x, y, z. Fig. 1 shows, by the range throughout which the curves of the various colours are identical, that, at large values of x, y, z the quantities become practically constant. The common factor takes account of the effects of dazzling (§ 32) of the eye. The quantities l, m, n are small constants, so that, if two of the component intensities, say y and z, are constant, the denominator contains a small term increasing in proportion to the varying intensity x. The factor has thus the same effect as the corresponding factor in the expression for σ, § 32 ; and it must be of the same type in the three variables. It accounts, if not applied under too great amounts of dazzle, for the upward trend of the curve in Fig. 1 under strong intensity ; that is, for the loss of perceptivity for small changes of intensity relatively to the value given by the unmodified law. It is the coincidence of the curves in Fig. 1 for all colours at high intensity which is the experimental basis for the form given to the factor. Helmholtz suggested a possible physiological cause for this effect in the intensive consumption of the arterial oxygen of the retina.

46. **Law of Intensity for Compound Colours.**—The above formulæ enable us to express the intensity of a compound colour of any definite composition. The composition being definite the ratios $x : y : z$ are definite, and so dx/x, dy/y, and dz/z have a common value, $d\lambda$ say. From the formula in § 45 we therefore find

$$dS=\frac{d\lambda}{1+lx+my+nz}\sqrt{X^2+Y^2+Z^2},$$

which is the general expression required.

In the special case of great intensity, since X, Y and Z then approximate to a constant value, k say, we have

$$dS=\frac{kd\lambda}{1+lx+my+nz}\sqrt{3},$$

which is in agreement with observation at high intensities provided that these do not produce too great a dazzling effect.

With regard to the general form of X, Y, Z there exists the guide given by the general form of the law for any given definite composition of the light. Apart from the dazzle factor the expressions in § 45 reduce to the type of § 32 if we put

$$\mathrm{X}=\frac{kx}{x_1+x},\quad \mathrm{Y}=\frac{ky}{y_1+y},\quad \mathrm{Z}=\frac{kz}{z_1+z},$$

so that x_1, y_1, z_1 represent mean values of the self light for the respective fundamental stimulations. Hence we have

$$d\mathrm{S}=\frac{k}{1+lx+my+nz}\sqrt{\left(\frac{x}{x_1+x}\right)^2+\left(\frac{y}{y_1+y}\right)^2+\left(\frac{z}{z_1+z}\right)^2},$$

where k is a constant.

The self light constant is smaller for blue light than for red and green lights, for which the values are about equal; and the term in which it has the smallest value is most effective as regards the magnitude of the factor, and therefore of the sensitivity. So the theory contains as a consequence the high sensitivity of the eye to colour variation in the blue region of the spectrum.

47. **Most Similar Colours.**—The difference as regards sensation between two given near colours, say those whose physiological stimuli are given in terms of the components x, y, z and $x+dx$, $y+dy$, $z+dz$ respectively may depend on the absolute intensity. That is to say, keeping the latter fixed, we may keep the colour of the former unaltered by maintaining the ratios of $x:y:z$ while we alter the magnitude of each component in the same ratio, say $(1+p):1$. By experiment we may adjust p so that the difference between the two colours is the least possible as tested by the eye. The two then form a pair of "most similar colours." The differences of their fundamental components are $dx-px$, $dy-py$, $dz-pz$; and so, by the formulæ of § 45, the square of the difference of sensation is

$$(d\mathrm{S})^2=\mathrm{X}^2\left(\frac{dx-px}{x}\right)^2+\mathrm{Y}^2\left(\frac{dy-py}{y}\right)^2+\mathrm{Z}^2\left(\frac{dz-pz}{z}\right)^2,$$

if we omit the dazzle factor.

The problem now is to determine p, so that $d\mathrm{S}$ may be a

minimum. We have therefore to differentiate the right-hand side with regard to p, and equate the result to zero. This gives

$$p(X^2+Y^2+Z^2)=X^2\frac{dx}{x}+Y^2\frac{dy}{y}+Z^2\frac{dz}{z},$$

and, inserting this value of p in the previous expression, we find

$$(X^2+Y^2+Z^2)(dS)^2=X^2Y^2\left(\frac{dx}{x}-\frac{dy}{y}\right)^2+Y^2Z^2\left(\frac{dy}{y}-\frac{dz}{z}\right)^2$$
$$+Z^2X^2\left(\frac{dz}{z}-\frac{dx}{x}\right)^2.$$

The result is quite independent of the units in which x, y and z are measured, and Helmholtz also points out that the only presumption involved is that X, Y, and Z are expressed relatively in the same units as x, y and z. In the special case in which the light is strong enough, X, Y and Z each have the constant value k, and we get

$$(dS)^2=\frac{k^2}{3}\left[\left(\frac{dx}{x}-\frac{dy}{y}\right)^2+\left(\frac{dy}{y}-\frac{dz}{z}\right)^2+\left(\frac{dz}{z}-\frac{dx}{x}\right)^2\right].$$

In either form the expression is that for the magnitude of the difference of colour sensation between two near colours of different colour tones which have been made as similar as possible to one another by suitable regulation of their brightness.

König and Brodhun made elaborate measurements of the perceptive power of the eye for the colour difference of near-lying spectrum colours. And they also made measurements on the mixture of spectrum colours by means of which the composition of spectrum colours in terms of three suitably chosen fundamentals could be found. In the former of these measurements they adjusted the two colours to apparent equality, and found for each pair the mean error of adjustment made in fifty trials. Helmholtz used these data to test the accuracy of his assumption that the trichromatic theory of Young could, by coupling it with the extension of Fechner's law of perception of brightness to the law of perception of colour, be applied to predict the trend of colour

perceptivity throughout the spectrum. And he did so with extraordinary success. Further discussion of the point will be made in Chapter X.

48. **Perceptivity for Intensity Difference and Colour Difference.**—For the purpose of comparing these perceptivities or sensitivities for brightness and colour, Helmholtz took the simplest form of Fechner's law in which the dazzle factor is unity and X, Y, Z each have the value k, so that

$$dS = k\sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2 + \left(\frac{dz}{z}\right)^2}.$$

In the case of brightness alone varying, each of the quantities in the brackets has a common value ε, say. Thus $dS = k\varepsilon\sqrt{3}$. The numerical value of k depends on the way in which the observations are made. Its value when deduced in accordance with the law of probability, when ε is taken as the mean error, is 1·8238 times as great as when ε denotes the smallest error which one can just perceive in ten cases. Uhthoff determined this ratio empirically, the values lying between 1·25 and 2·44, with a mean value 2·025, which Helmholtz regarded as in sufficient agreement with the theoretical value.

The smallest recognizable fractional difference of brightness with white light observed in König's measurements, with similar external arrangements, size of field of view, etc., as in the colour comparisons amounted to 0·0173. The equation $dS = k\varepsilon\sqrt{3}$ then becomes $dS = k(0{\cdot}0173)\sqrt{3}$. But the value of k is 1·8238 times larger in the observations on colour. Therefore, using the same unit for dS in both cases, the value of dS in the colour measurement should be $(0{\cdot}0173)\sqrt{3}/1{\cdot}8238 = 0{\cdot}01643$. The mean value actually found from König's observations was 0·0176.

Helmholtz remarks that "this agreement can well be denoted, under the given conditions, as beyond expectation. It corresponds to the assumption from which we have here proceeded that the perception of colour differences originally rested on the perception of difference of brightness." He adds that a further proof of the law here expressed would be better carried out by direct mixing of two spectrum colours

in different ratios, in which the mixture ratio can be read directly on the apparatus, and in which also more diverse comparisons are established than enter in between directly neighbouring colours.

49. **Shortest Colour Lines.**—The next great step taken by Helmholtz in the explanation of colour vision phenomena dealt with the passage by some continuous process from one colour to another. He pointed out that the condition

$$(dS)^2=(dS_1)^2+(dS_2)^2+(dS_3)^2$$

makes it possible to consider, out of all the arbitrarily variable paths by which a colour could be made to change continuously from some initial value to some other finite value, that one path in which the sum of all the successive perceptible differences has the smallest value. This series of intermediate colours he calls a *Shortest Colour Series.* And he indicates that it is of the essence of a shortest colour series, that with such colours as show equally great difference from an end colour, that one lying in the shortest colour series would also seem more similar to the other end colour than all the neighbouring colours.

If we represent the component sensations S_1, S_2, S_3 by lengths measured along three mutually perpendicular axes, the above equation shows that the shortest line joining an initial and a final point is the straight line joining the two. For it states that the square of an elementary step in sensation is the sum of the squares of its constituents: and two such successive steps must be in the same direction if the step is to be the shortest possible. But we desire to express the sensation in terms of x, y and z rather than in terms of S_1, S_2, S_3; for these represent the stimuli which originate the sensation. We must therefore replace dS_1, dS_2 and dS_3 by their equivalents in terms of the stimuli.

To obtain expressions for the equivalents sufficient for this purpose, Helmholtz took Fechner's form of the relation "which agrees with observation throughout an extraordinary stretch of brightness," and so put $dS_1=dx/(a+x)$, $dS_2=dy/(b+y)$, $dS_3=dz/(c+z)$, where a, b, c represent the components of the external stimuli which are equivalent to the components of "self-light" in their physiological action. He remarks:

"I have advanced this as a probable hypothesis." Thus we have

$$(dS)^2=\left(\frac{dx}{a+x}\right)^2+\left(\frac{dy}{b+y}\right)^2+\left(\frac{dz}{c+z}\right)^2.$$

It is obvious that, in the x, y, z construction, the shortest colour series cannot be straight lines; for that would require

$$(dS)^2=(dx)^2+(dy)^2+(dz)^2.$$

But the ratio $dx/(a+x)$ is equal to $d(\log[a+x])$, and so on: so that if we put $\log(a+x)=\xi$, $\log(b+y)=\eta$, $\log(c+z)=\zeta$, we have

$$(dS)^2=(d\xi)^2+(d\eta)^2+(d\zeta)^2.$$

That is to say, if, instead of using x, y, z as co-ordinate quantities, we use $\log(a+x)$, $\log(b+y)$, $\log(c+z)$, the shortest colour series would be straight lines. From this fact we can easily judge the manner in which, in the colour pyramid, the shortest colour lines deviate from rectilinearity.

The equation of a straight line in the ξ, η, ζ field, when transformed by the relations connecting ξ, η, ζ with x, y and z, become the equations of the same locus in the x, y, z field. But the equations of a straight line passing through the points denoted by the suffixes 1 and 2, are

$$\frac{\xi-\xi_1}{\xi_2-\xi_1}=\frac{\eta-\eta_1}{\eta_2-\eta_1}=\frac{\zeta-\zeta_1}{\zeta_2-\zeta_1}.$$

Substituting the values $\xi=\log(a+x)$, etc., we get

$$\left(\frac{a+x}{a+x_1}\right)^{\xi_2-\xi_1}=\left(\frac{b+y}{b+y_1}\right)^{\eta_2-\eta_1}=\left(\frac{c+z}{c+z_1}\right)^{\zeta_2-\zeta_1}.$$

Each one of ξ, η, ζ may be positive, zero, or negative; but, on the Young-Helmholtz view, no one of x, y, z can be negative. Nor can any of a, b, c be negative. Through a large range of intensities they can, as we have seen, be regarded as positive constants. Observations by Burch (§ 109) indicate that their values may become very small under prolonged rest of the eye in darkness. In any case the geometrical relations of the shortest colour lines become clearer by consideration of the possibility that a numerator or a denominator within any of the brackets may be zero. This corresponds

to total absence of all light including the self light of the eye.

The null point of all light may be indicated by the symbol 0, just as the symbols 1 and 2 denote definite points in the colour field. A line drawn through 0 parallel to the x axis corresponds to the vanishing of $b+y$ and $c+z$ whatever positive value $a+x$ may have, and so on.

50. **Rectilinear Shortest Series.**—The simplest cases of the shortest colour series occur if these lie in one direction. There are two ways in which this condition may arise.

If the two specified points 1 and 2 have the same projection on one of the co-ordinate axes, say the x axis; that is, if the two colours indicated by these points have the same x stimulation, we have $\xi_2=\xi_1$. In this case, unless numerator or denominator in the first bracket above vanished, the expressions involving y and z respectively are equal to each other whatever be the value of x. But there is more restriction, for the expression involving x is necessarily unity when $\xi_2=\xi_1$; therefore we have necessarily $y=y_1$, and $z=z_1$, independently of the value of x. So in this case any line parallel to the x axis is the locus of a shortest colour series between any two points lying on it.

Similarly, lines parallel to the y or z axes are shortest colour lines. They are followed when a fundamental colour is added to white.

Again, if $\xi_2-\xi_1=\eta_2-\eta_1=\zeta_2-\zeta_1$, we have

$$\frac{a+x}{a+x_1}=\frac{b+y}{b+y_1}=\frac{c+z}{c+z_1}.$$

The first of these equations represents a plane passing through 0 and perpendicular to the x, y plane since it does not involve z. The second similarly represents a plane through 0 and perpendicular to the y, z plane. Therefore the straight line of intersection of these planes is a line of a shortest colour series. So any straight line through 0 is a shortest colour line between any pair of points lying on it.

51. **Plane Shortest Series.**—The next simplest case is that in which the series though not rectilinear nevertheless lies in one plane. This can be the case if the curve has the same form with reference to two of the co-ordinates, say,

x and y. The condition occurs if $\xi_2 - \xi_1 = \eta_2 - \eta_1$. For then, whatever be the value of z, we have

$$\frac{a+x}{a+x_1} = \frac{b+y}{b+y_1},$$

the equation of a plane passing through an axis drawn through 0 parallel to the z axis. Similarly, under the conditions $\eta_2 - \eta_1 = \zeta_2 - \zeta_1$ and $\zeta_2 - \zeta_1 = \xi_2 - \xi_1$, planes passing through the other principal axes drawn through 0 may contain shortest colour series. That is to say, if two given colour points lie in one plane passing through a co-ordinate axis of the colour pyramid, the shortest colour series joining them lie in that plane. Any such plane passes through the null point, 0, of all light objective or subjective; for both numerators must vanish if one vanishes.

If we call ε the null point for external stimulation alone, that is the apex of the colour pyramid where x, y, and z are all zero, all lines of constant external excitation are straight lines passing through ε. But, as in § 50, these are not lines of shortest colour series unless they also pass through 0. Only the single line which passes through the null point of all stimulation and the null point of external stimulation can satisfy both conditions. In all other cases lines of constant quality of external stimulation are not lines of constant colour perception. As the external stimulation increases or decreases in intensity, without changes in the proportions of the fundamental stimulations, the colour apparent to the eye must change in consequence of the presence of self light, supposed for simplicity to be constant in amount and quality. Even with that restriction, the change in colour in passing from ordinary to very feeble, or from ordinary to very intense, illumination is in agreement with observation.

With increasing brightness all spectrum colours, any one of which possesses a fixed relative composition with regard to the fundamentals and so lies on a straight line passing through ε, becomes more similar to white of a yellowish tinge. The most rapid passages thereto occur in the change of green into yellow, and of violet into whitish blue. Greater increase of intensity is required to change blue into white and red into yellow. The only colour which remains markedly unchanged at all intensities

is a yellow white. This indicates that the colour tone of the line 0ε is a yellowish white. Since the deviation of that line (called by Helmholtz the Principal Line of the colour system) from the line of pure white, equally inclined to the three axes, is due to Fechner's self-light constants a, b, c, this yellowish white must be regarded as the colour tone of the self light.

This colour tone of the self light is superposed on all colour tones belonging to the external light. Therefore, to maintain a constant colour tone with increasing intensity, the tendency of the externally caused colour to become white must be checked by taking it of more and more saturated value. That is to say, the curves of constant colour tone must bend away from the line of white sensation, the line through ε equally inclined to the three fundamental axes as above stated. Ultimately the tendency of all colours towards white at very high intensities would swamp the effect of the self light, and even yellow would approximate to white.

52. **Projections of the Curved Shortest Lines passing through the Absolute Null Point.**—Any one of the three equations given last in § 49, is the equation of the projection of the tortuous shortest colour line on one of the fundamental planes. Thus

$$\frac{a+x}{a+x_1}=\left(\frac{b+y}{b+y_1}\right)^{\frac{\eta_2-\eta_1}{\xi_2-\xi_1}}$$

is the projection on the x, y (red-green say) plane of the shortest colour line on which the colours 1 and 2 lie. It evidently passes through the null point 0, provided that $\eta_2-\eta_1$ and $\xi_2-\xi_1$ have the same sign; for both numerators must then vanish if one does so.

Now we have

$$\frac{\eta_2-\eta_1}{\xi_2-\xi_1}=\frac{\log\dfrac{b+y_2}{b+y_1}}{\log\dfrac{a+x_2}{a+x_1}}.$$

Hence if $y_2>y_1$ and $x_2>x_1$, or conversely for both, the fraction is positive. In other words, if the slope of the line joining the projections of the points 1 and 2 on the x, y plane is positive, the projection of the curve passes through 0. Con-

versely, if the projection passes through 0, the slope of the projection must always be positive.

The ratio of the logarithms is unity if y_1 and y_2 bear to b the same ratios as x_1 and x_2 bear to a. This occurs only if the projection is that of the Principal Line. If the ratio of the logarithms is greater than unity, the first equation shows that $a+x$ increases at a greater rate than $b+y$; so that the projection is then concave to the parallel to the x axis drawn through 0. Conversely, if the ratio of the logarithms

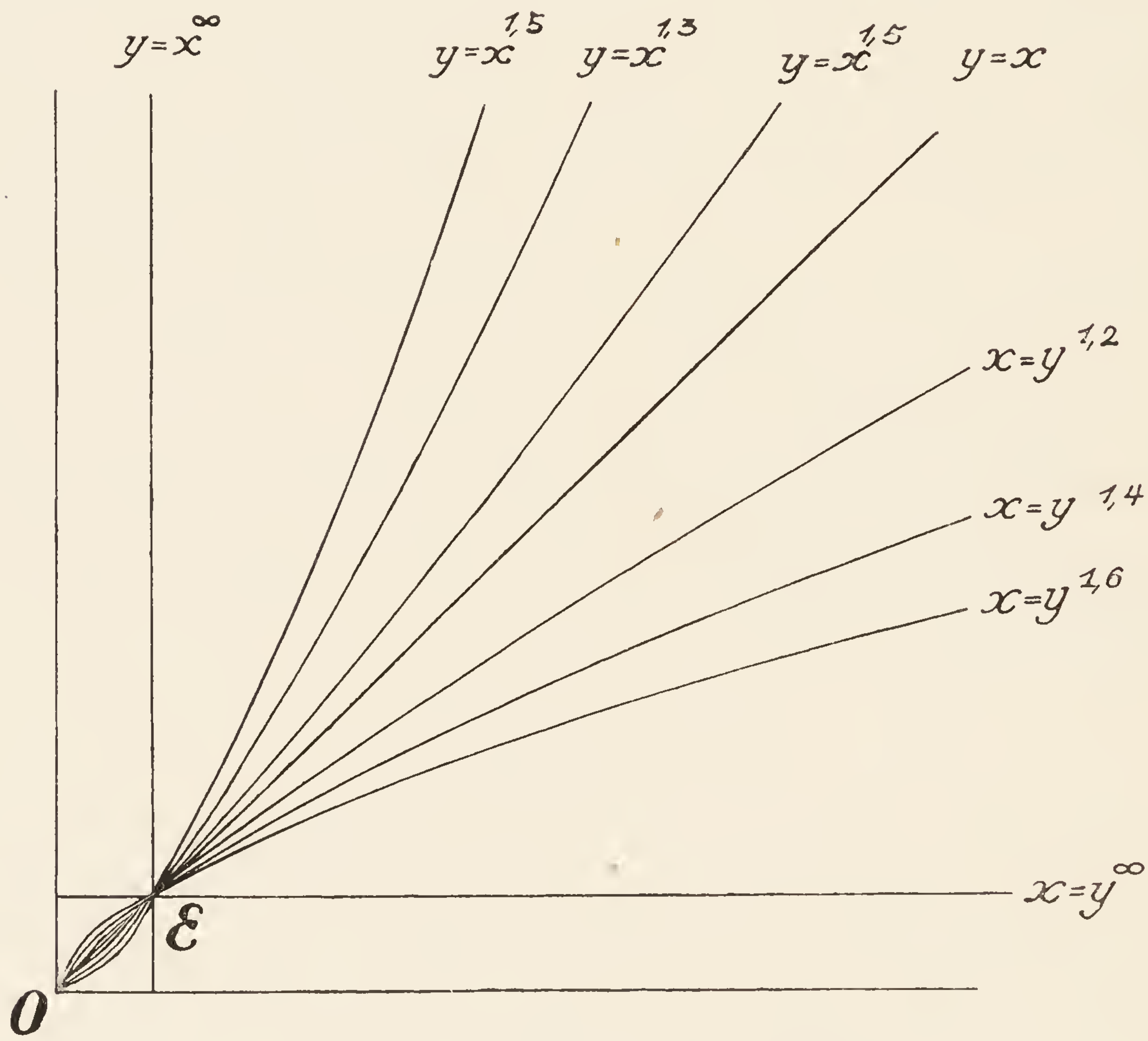

Fig. 10.—Shortest Colour Lines.

is less than unity, the projection is concave to the parallel to the y axis drawn through 0. The change from the one condition to the other occurs when the curvature vanishes, that is, in the projection of the Principal Line.

Further, if we take the point 1 as ε, that is, put $x_1=0$, $y_1=0$, the first equation shows that both x and y must be zero if one of them is zero. Therefore all the projections pass through the projection of the point ε as well as through 0.

In Fig. 10, given by Helmholtz, a bundle of these projec-

tions is shown for values of the ratio of $\eta_2-\eta_1$ to $\xi_2-\xi_1$ expressed at the margin.

By symmetry, the projections on the other fundamental planes are obtained through interchange cyclically of x, y, z and a, b, c.

Thus all the actual tortuous curves in the colour pyramid pass through 0 and ε, and bend in the manner indicated by these three sets of projections.

53. **Projections of the Curved Shortest Lines not passing through the Absolute Null Point.**—If, in the first equation of § 52, $\eta_2-\eta_1$ and $\xi_2-\xi_1$ have opposite signs, the equation becomes

$$\frac{a+x}{a+x_1}=\left(\frac{b+y_1}{b+y}\right)^{\frac{\eta_1-\eta_2}{\xi_2-\xi_1}}$$

where the exponent is again positive. But now the numerator $b+y_1$ can never be less than b since the point 1 is a point of externally caused colour. Therefore $a+x$ can never vanish unless $b+y$ is infinite. So these curves asymptotically approach parallels to the x and y axes drawn through the point 0. The only portions of them which have value relatively to objective light are the portions lying in the region of x and y positive: indeed the portions in the immediate neighbourhood of the spectrum curve. The complete curves resemble rectangular hyperbolas, which they indeed become in the particular cases where $\eta_1-\eta_2=\xi_2-\xi_1$. Similar projections appear on the y, z and z, x planes. Though the quantities $\xi_2-\xi_1$, $\eta_2-\eta_1$, $\zeta_2-\zeta_1$ may each be positive or negative independently, their ratios must all be positive unless two are negative. Therefore, in the case of any two given points 1 and 2, the three plane projections must all pass through 0, or only one of them does so.

54. **Colour Change with Constant Ratios of Fundamentals.**—The nature of these projections on the fundamental planes, and the distribution of the plane curves, makes easy a discrimination of the manner of change of colours as the intensity of the light is varied. The Principal Line is the only curve of shortest series which is also a line of constant mixture ratios. Every plane through a co-ordinate axis and the principal line contains plane curves which are convex to the

principal line, trending towards the co-ordinate axis on the one side, and towards the line at right angles to it on the other. Thus, if the x axis be the fundamental axis, all intermediate shortest series lines lying in the plane passing through the x axis and the principal line trend from the principal line towards the x axis, and all shortest series lines in this plane, on the remote side of the principal line, trend from the principal line and towards the line in which the plane cuts the y, z plane.

Helmholtz called the colour characteristic of the latter line the Principal Counter Colour. In combination, when properly proportioned, with the corresponding fundamental colour to which it is "counter," it produces the self-light colour, the colour of the principal line. The counter colours are analogous to the colours which are complementary to the fundamental colours ; the difference being that the principal complementary colours produce white, whereas the principal counter colours produce the yellowish white of the self light when they are mingled with the fundamentals in proper proportions. "If carmine-red, ultramarine blue, and blade-green correspond in colour tone to the fundamental colours, and yellow white to the principal colour, approximately verdigris, yellow and purple would be the principal counter colours."

Seeing that the self light is always present in eye observations, it is in part with respect to the fundamental colours and their counter colours, and not entirely to them and their complementary colours, that the apices of the colour hexagon (§ 42), in the case of so-called hexachromy, would be manifest. But complementariness is also practically determined in presence of self light ; so that the practical white is affected by the colour of the principal line, and the principal counter colours are in part efficient in the practical complementary colours of eye observations. The proportion to which it avails is given by the Newtonian law of colour mixture. So long as the external illumination overbalances that of the self light, the effect is negligible, and observation deals practically with the fundamentals and their complementaries.

All colours in the three planes passing through the axes and the principal line can be compounded of the self light colour and the three fundamentals on the one hand or their three counter colours on the other. But the self-light colour can,

as we have seen, be compounded of any one of the fundamentals together with its counter colour. Therefore any of the colours in these planes can be compounded of the three fundamentals and the three principal counter colours. Similarly, colours in the angle between any pair of the planes can be compounded of colours in the planes ; so all colours in the field of vision can be compounded of these six colours, as in so-called hexachromy. But each counter colour can be compounded of the two fundamental colours in the plane of which it lies : so that the whole colour scheme is trichromatic. Reference being made to the absolute fundamentals, only the three apices can appear in the colour diagram. The other apices can only appear when the actual fundamentals of a particular eye are "derived" from the absolute fundamentals, and that only under presumption of suitable derivation (§ 42).

Since each shortest series line in the field of vision which passes through ε, apart from the straight lines parallel to the axes, the principal line itself, and the curves in the three principal planes passing through the principal line, have curved projections on the three principal planes ; and since all these projections as they proceed outwards from ε, bend away from the projections of the principal line and approach the fundamental co-ordinate axes, we see that any straight line drawn through ε, i.e., any line of constant proportions of fundamental colour mixture must, as it passes outwards to regions of higher intensity, cross curves of shortest colour series, continuously reaching those which differ less and less in colour from the self-light line. Therefore all spectrum colours tend, at sufficiently high intensity, to become yellowish white.

The fundamental colours alone, with their counter colours in so far as the effect of self light is concerned, do not change with increasing intensity. But any spectrum colour which has the hue of a fundamental colour with admixture of white, e.g., light blue, must belong to a straight line through ε which cuts shortest colour series, and so will become whiter with increasing intensity. The addition of white to blue is more effective than that of white to yellow, for the blue lies nearer to the white line than to the principal line. The stretch of colour from fundamental red to purple, and that from yellow

green to verdigris green, being more remote from white than from the principal line, are changed, on the contrary, to whiter and yellower colours.

All these conditions are well known, as Helmholtz pointed out ; but it is almost startling to find them occurring as simple direct consequences of the trichromatic theory with the inclusion of the phenomena of self light as dependent on quantities of a type introduced quite independently by Fechner in the consideration of phenomena of intensity alone without consideration of colour. Just as it was an intuitive perception of genius which led Young to the trichromatic theory of colour vision, unattained by Helmholtz in his own endeavours, so it was an intuitive perception of genius which led Helmholtz to the simple extension of Fechner's law so as to bring colour phenomena within its sweep. It is such facts as those here given which stamp a theory with the seal of truth, and turn it into a broader fact than those which it includes. But further triumphs lie within its power.

54A. **Colour Changes in Extremely Weak Light.**—The shortest series lines which lie between a principal counter-colour line and the fundamental colours of which it is compounded are all convex to the counter-colour line and so approximate to the fundamental colour nearest them as the intensity of the external light increases. Thus spectrum colours which are in the neighbourhood of counter colours, when diminishing in intensity, without change of fundamental proportions, change through the shortest colour series which approximate more and more to the nearest fundamental as the intensity becomes very weak. This vanishing of the counter colours in the spectrum, yellow and green-blue, under feeble intensity wherein the stronger Fraunhofer's lines were yet visible, was first observed by von Bezold and Brücke. Red, green, and blue still appeared as colours, and were therefore considered to be the fundamental physiological colours. Von Bezold and Brücke held the view that an element of perception in a compound perception was ineffective in the latter if it did not attain the threshold value (§§ 45, 60, 61) characteristic of it by itself.

The first light which a radiating body emits as its temperature rises is " dark fog grey," then " ash grey," then yellowish

grey. It is in fact the yellowish-green light of the spectrum which first appears. This is the converse phenomenon to the one just considered, in which the intermediate colours, yellow and green-blue, first vanish from the spectrum in growing feebleness of the light, "the places occupied by them being shared by red, green, and violet-blue." In still weaker illumination, these colours become red-brown, olive-brown, and blue-grey respectively. Finally all colour vanishes in extremely weak illumination, red last of all, each kind, when it has fallen below the threshold of perception, appearing equally grey.

The spectrum then seems colourless with the maximum of intensity in the green at about the wavelength 535 micromillimetres: and the intensity curve or luminosity curve, agrees fairly with that of monochromatic eyes.

Red glow occurs later as the temperature of the body rises, but the spectrum then extends between the Fraunhofer lines B and *b* in the red and green respectively. Helmholtz ascribes the earlier appearance of green light to the greater sensitivity of the eye for these rays. He adds, "One can make similar observations on the so-called phosphorescent clouds which sometimes are visible in a very clear sky in the height of summer in the north, and seem to be extraordinarily high-lying little clouds which still have the twilight from the sun below the horizon. They seem grey green, their light is lost through a red glass, whilst other objects as highly illuminated by distant gaslight remain visible through the red glass. Thus gaslight contains more red rays than that of the phosphorescent clouds even although the latter is apparently the twilight of the sun even farther down than the horizon corresponding to their position. The explanation of this phenomenon is furnished by the above given relation of the spectrum colours to the fundamental colours. Each spectrum colour excites to a moderate extent all three fundamental colours. We perceive light if we can discriminate the total light from darkness, that is, if the single excitation steps over the threshold. In order to discriminate colour, we must distinguish smaller amounts of the fundamental colours in one mixture from greater amounts in another."

In terms of the fundamentals R, G, V finally chosen by

Helmholtz the following are the relative stimulations contained in spectrum red and violet, and in white :—

	Red.	Violet.	White.
R. . . .	0·6093 ..	0·3528 ..	0·3333
G. . . .	0·1998 ..	0·2498 ..	0·3333
V. . . .	0·1913 ..	0·3973 ..	0·3333

In connection with these data he remarks that König's observations show that the brightnesses at which a change of 0·6 in their magnitudes can be distinguished are from eight to twenty times greater than the threshold values. " But the lights which are in the former ratio to each other must be distinguishable from each other, if spectrum red of that strength is to be capable of distinction from white. In the other colours the ratios are nearer unity and with more difficulty distinguished from it."

55. **Deviations from the Newtonian Law of Colour Mixture.**—If the Newtonian law were strictly correct, all shortest colour series would be straight lines passing through ε. The deviations from that condition therefore indicate deviations from the strict sequences of that law. Helmholtz continues : " The law of the undisturbed superposition of the elementary excitations is evidently a very accurate expression for a wide region of phenomena, and was completely verified in relation to the older method of experiment, particularly where one used for the investigation the weaker colour contrasts and small luminosities of pigment colours. Very small departures are not capable of certain proof in these on account of the fact that small changes in the composition of the illuminating light are able to change somewhat the colour equations. But the newer accurate measurements with spectrum colours seem nevertheless to show that the sufficiency of Newton's law is not unlimited.

" The first observations of this kind were made on dichromatic eyes by Preyer, A. König, and van der Weyde. They showed that the spectrum colours which seem white in very weak illumination seem to be yellower in greater brightness, and must be compensated by blue light in order to remain equal to the white. E. Brodhun (green blind) then made colour equations between a single middle colour and two others under A. König's directions, and likewise found that with

increasing intensity more of the warmer colour must be taken in order to reinstate the equation. The changes of light intensity were made in different ways in order to exclude any change of wavelength of the comparison colour, partly through altered position of the Nicol, partly through widening of the eyeslit, partly through widening of the collimator slits. This widening of the slit took place, conformably to the design of the apparatus, equally on both sides.

"Indications of such changes were also observed for trichromatic eyes by Albert and A. König. But the latter only found that the difference of saturation between pure or almost pure yellow colour tones and the mixed tones seemed more distinct in weaker brightness.

"From the great changeability of the light perceptive substances of the retina . . . a change of their photochemical decomposability with regard to different wavelengths seems not impossible. If we make the simplest assumption, from which we have already deduced the existence of anomalous trichromasy, that a mixture of two different photochemical substances may be present in the retinal ends of the optic nerve fibres, and that of these two the one most quickly decomposed by light is restored relatively slower, intensive light would be able to alter the mixture ratio and therewith also the form of the sensation curve. In the dichromats in whom the one curve is certainly changed, and transition between the two extreme forms seems to occur, such a greater changeability of the mixture becomes probable.

"One can imagine that in a group of retinal elements the green perceptive substance becomes regularly transferred into the red perceptive, that in single individuals the latter is entirely or almost entirely inactive. These would be red blind. If in other individuals the other group of the retinal elements, which should be closed to the inversion, were drawn completely or incompletely into the change, green blindness or anomalous trichromasy would occur. If strong light action changed a small part of the red perceptive substance into the green perceptive, before this is destroyed, the maximum of perceptibility would approach the green, and thereby the neutral point of the dichromatic spectrum would be displaced towards the blue.

"I give this quite hypothetical view here only in order to show that the facts quoted by no means abrogate Newton's law. They only show that it, like very many other laws of nature, gives a first approximation, always very good and extending throughout wide limits, to the full truth."

CHAPTER VII

DICHROMASY AND MONOCHROMASY

56. **Vanishing Colour Areas.**—The essence of trichromatic vision, whether normal or abnormal, is that the derived colour triangle, R′G′B′, shall have a finite area.

It is to be remembered that, whereas in the absolute equilateral triangle the corners represent the localities of unit amounts of the fundamental stimuli, in the derived triangle, the strengths of the derived fundamentals need not be equal. There are, in fact, two ways in which the derived conditions may present peculiarities: first, through the localization, i.e., the quality of the new fundamentals; second, through their relative magnitudes.

There is thus an eight-fold infinity of possibilities, corresponding to the freedom of choice in each of the three derived fundamentals, of the two ratios of the absolute fundamentals therein present, and to the two magnitude ratios of these three derived quantities. Dichromasy occurs when the three derived fundamentals lie in one line.

In the development of the subject of dichromasy, strict scientific procedure led to the postulation first of the simplest possible condition in order that it might be tested by observation; no more complex condition being assumed until observation gave cause. This first step was taken by Young himself, and he was followed by others, including Helmholtz. The simplest assumption is that two of the absolute fundamentals are unaltered, while the magnitude of the third vanishes. The next simplest is that one is unaltered while two are fused, and so are represented by a single point in the diagram. And so on.

Helmholtz actually developed the general theory of dichromasy. Yet remarks made by many critics of the trichromatic

theory, with its associated dichromatic theory, seem to indicate entire ignorance of the possibilities. It is well therefore to give his words.

"The mode of explanation adopted by Th. Young and most of the workers at the theory, and formerly taken up by myself, by E. Hering, A. König, and C. Dieterici, that in dichromasy one of the fundamental excitations simply does not exist is contradictory to the result mentioned [yellow-blue vision]. But there is a general theory possible regarding the nature of dichromasy, in which it ceases to be necessary that the failing colour should be a fundamental colour, and yet the rule remains that all pairs of colours which seem identical to the normal trichromatic eye also seem identical to the dichromatic eye.

"To make this clear by a simple example, assume that the action of light which otherwise excites the perception of green does not excite the green perception nerve, but affects the red and blue perceptions in a definite fixed ratio. All perceptions of such an eye would seem to be compounded of red and blue; it would be dichromatic. But the colours which in the colour diagram lie on that straight line which is drawn through the position of the green perception, would in general not seem alike, as would be the case under the older assumption where simple lapse of the green sensation was assumed. For, instead of the varying amount of green in the trichromatic eye, a varying amount of a definite purple colour would here be added to the already present different mixed purple, and would alter it in the majority of cases. In fact, the intersection point of those lines of the dichromatic field which contain like seeming colours, would lie outside the colour triangle beyond the green corner.

"This condition remains unaltered if we yet further assume that each excitation of the red, even that just considered, excites also the green perceptive parts of the nerve in a definite proportion, and so would bring a definite kind of yellow into perception, and each excitation of the blue similarly a definite kind of green-blue. Then all perceptions of such an eye would be compounded of yellow and green-blue, whilst the intersection point of the dichromatic lines of like appearance would not thereby be changed."

The conditions referred to are illustrated in projection in

Fig. 11, O, the origin, being situated at the apex of the colour pyramid, and OR, OG, OB being equal lengths along the three co-ordinate axes, while the length of the perpendicular drawn from a corner of the colour triangle RGB to the opposite side is unity. If the red and blue sensations work normally while the green is entirely ineffective, all colours in the normal diagram situated on a line GH meeting RB in H, present to the abnormal eye one colour alone, a purple containing red and blue in the proportions HB to HR. But if the green stimulation, while ineffective as regards the green sensation, stimulates the red sensation in the proportion $p : 1$, we may represent this by taking a point M, on the parallel through G to RB, such that MN drawn perpendicular to GB is equal to p, and drawing through M the line AMB.

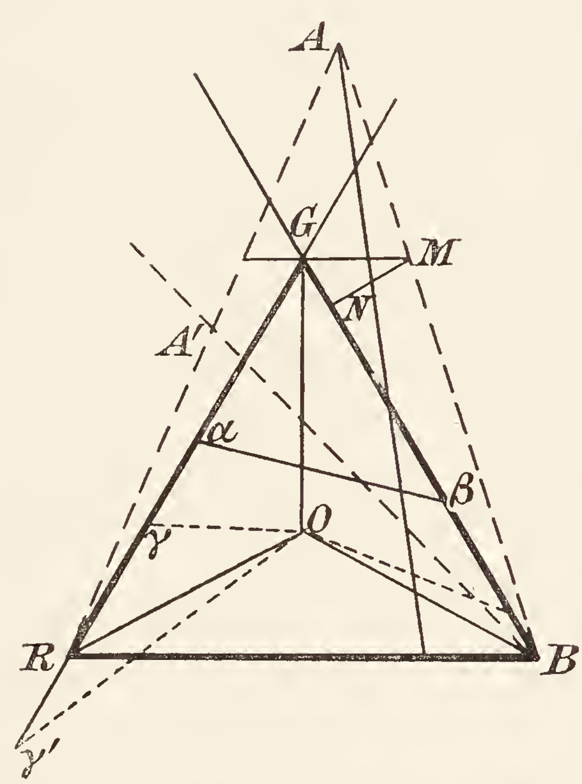

Fig. 11.—The Dichromatic Colour Triangle.

Any point on the line AB corresponds to no red stimulation of the abnormal eye. For the perpendicular from that point to GB represents at once the positive red stimulation communicated through the green nerve centre and an equal negative stimulation communicated directly through the red centre. Of course no such point has any physical reality on the Young-Helmholtz theory, but the construction gives the proper results for the regions within the colour diagram where stimulations are positive.

In the same way, a quantity q replacing p, we find a line AR giving the locus of entire absence of blue stimulation in this abnormal eye. Since the eye possesses only red and blue sensitiveness, the point A represents the locality of entire absence of stimulation under external illumination, which obviously must lie outside the normal colour triangle. The point P, between R and B, with $PB/PR = p/q$, represents the position of the third fundamental, linear in terms of the other

two, due to the green stimulation. The line Oγ, perpendicular to the plane AOB, represents the resultant of the red stimulations, unity along OR and p along OG. A line such as AH represents a locus of constant colour as seen by the abnormal eye, just as GH did when G represented the failing colour.

If the total red stimulation excited the green sensation proportionally, so that the resultant lay at α; and similarly if the total blue stimulation affected the green sensation proportionately so that the resultant lay at β; the triangular area αGβ would be the only effective portion of the triangle RGB; and the lines Aα, Aβ would form the boundaries of the bundle of lines radiating from A of which each corresponds to a single colour, Aα bounding the set on the yellow side while Aβ bounds it on the green-blue side. Colours formerly included within the angles RAα and βAB have disappeared from perception.

In the former case, in which RPB represents all the possible stimuli, the line may be regarded as the limit of a colour triangle in which the apex P has approached indefinitely near to the base RB. But if we may consider, as in Chapter V, that an apex of a derived triangle may lie outside the absolute colour triangle, we have, from this point of view, to consider the possibility of P lying on the base but external to the limited range RB. The lines Oγ and BA would in that case take positions such as Oγ', BA$'$. The construction indicates that none of the stimulations corresponding to those points in the triangle of normal vision which lie above the straight line joining A$'$ and B are effective in relation to the dichromatic eye. If both stimulations, red and blue, communicated through the centre which normally gives rise to the green sensation, are negative, A$'$ lies within the normal triangle and P lies within the range RB. The triangle RA$'$B might then be regarded as the triangle of dichromatic perception, with the proviso that all points on any one line drawn from A$'$ to the base indicate the same colour impression. But, if we are to maintain the direct purpose in the use of the colour triangle, i.e., the representation of colour difference, the proper dichromatic region is the linear extent of the collapsed triangle RPB.

Dichromasy in general is due to the change of one, or two,

or all of the normal fundamentals so that the three derived fundamentals lie in one straight line. Part of the line joining the two extreme fundamental points must lie in or on the normal colour triangle : but there is no a priori reason why all the derived fundamentals should not lie outside the normal triangle. The only requisite condition on the Young-Helmholtz trichromatic theory is that no region corresponding to a negative value of any one of the absolute or the derived fundamentals can correspond to actual vision. Whether a derived fundamental lies or does not lie outside the absolute triangle is a point to be settled by observation alone, just as is the question whether or not a definite invariable absolute triangle exists.

57. **Vanishing Colour Lines. Colour Points.**—The three fundamentals on the dichromatic colour line may be situated very close together, and may even be absolutely coincident. Two of the three colour freedoms then vanish, and the condition of monochromasy ensues. This condition may result although the three coalescing points are not collinear. The derived triangle may retain typical triangular form, its linear dimensions tending to zero.

So long as a finite threshold stimulation is required, the linear dimensions need not actually vanish in order that practical monochromasy may be experienced. A minimum departure from coincidence is sufficient. Similarly, a minimum departure from collinearity of fundamentals is all that is necessary to ensure practical dichromasy.

Indeed, on Helmholtz's hypothesis, absolute dichromasy and, still more so, absolute monochromasy may be extremely improbable. When the average threshold values for excitation of the fundamental colour impressions are known, the percentage of occurrences of practical monochromasy and practical dichromasy become calculable on the assumption that there is no physiological or anatomical weighting of particular conditions.

CHAPTER VIII

SENSATION AND LUMINOSITY

58. **Apparent Luminosity.**—The general meaning of the term luminosity or brightness requires no special discussion. For qualitative purposes its general scientific use coincides with its ordinary everyday use : and this may be asserted even when one contrasts two differently coloured lights, saying, for example, that a given blue light is strong and that a particular red light is weak. But, in the comparison of the strengths of two differently coloured lights, the necessity for more definite discussion arises.

The actual physical intensities of the lights do not form here the matter under consideration. These are simply measured in terms of the total energy of the light incident normally per unit area of a surface in unit time. It is the magnitude of the sensation of brightness that arises in eye observations. And the magnitude of the sensation depends upon the magnitude of the physiological stimulus, which again depends on the physical intensity of the light as just specified. Further, the question of judgment, even in the case of two similar normal eyes, has undoubted importance. The intensity of "colour glow," evident, for example, in the extreme red or the extreme violet light of the spectrum where the luminosity is feeble, has an effect difficult to eliminate. In Helmholtz's words, "the light quanta of two different coloured lights admit of no general valid quantitative comparison by the eye." "I scarcely trust my judgment regarding the equality of brightness of different colours, at least as to greater or smaller in extreme cases. But I admit that it is possible to darken one of two coloured fields so that there is no doubt that the other is brighter." "I have the feeling that, in equating luminosities of different colours, the question is not that of comparing one-

fold magnitudes but two-fold magnitudes compounded of brightness and colour glow, for which I cannot give any simple summation, and which I cannot scientifically define further." The difficulty is universally admitted.

Nevertheless it is found to be possible to obtain consistent estimates of heterochromic luminosities by means of eye observations. It is essential that the relation of colour and brightness should be made evident on the basis of theory.

59. **Experimental Determination of Equal Luminosities.**—Since any compound colour whatsoever can either be matched by a definite spectrum colour, lightened by the addition of white, or darkened, or, in the case of the ultra spectrum colours such as purple, can be matched by means of lights from the two extreme ends of the spectrum, the purely scientific problem is confined essentially to the case of the spectrum. The measurements are made definite by comparison of a coloured spectrum light with the "white" light from which the spectrum is produced. The intensity of the comparison white can be varied until the two lights seem to be equally luminous. The luminosity of the white is known, because the amount of its reduction is known, and is taken as the measure of the luminosity of the coloured light. This, being a direct eye test of equality of brightness, is subject to all the adverse influences entering into an act of judgment.

The other chief method of experiment consists in intermitting the two lights under comparison at a regular rapid rate. The intermission is produced by means of a rotating disc with alternate open and closed sectors. Flicker is observed when the speed is not too great. It is found that the rate of rotation at which the flicker ceases is a minimum when the coloured and uncoloured lights appear to the eye to have the same luminosity. The two methods should therefore give the same results, but there should be, and there is, a smaller limit of error in observations made by the flicker process since the question of judgment of equality, in one visual respect, between two things which differ visually in another does not enter. When the slight differences of luminosity throughout the spectrum between lights of near wavelengths are observed, so that difference of colour does not practically arise, the actual luminosities are obtainable by summation and are found to

agree practically with those given by the flicker method if the illuminated areas are not too large. Ives' results agree completely when a necessary correction is applied.

Abney has shown that the sum of the luminosities of the spectrum colours measured separately is equal to the luminosity

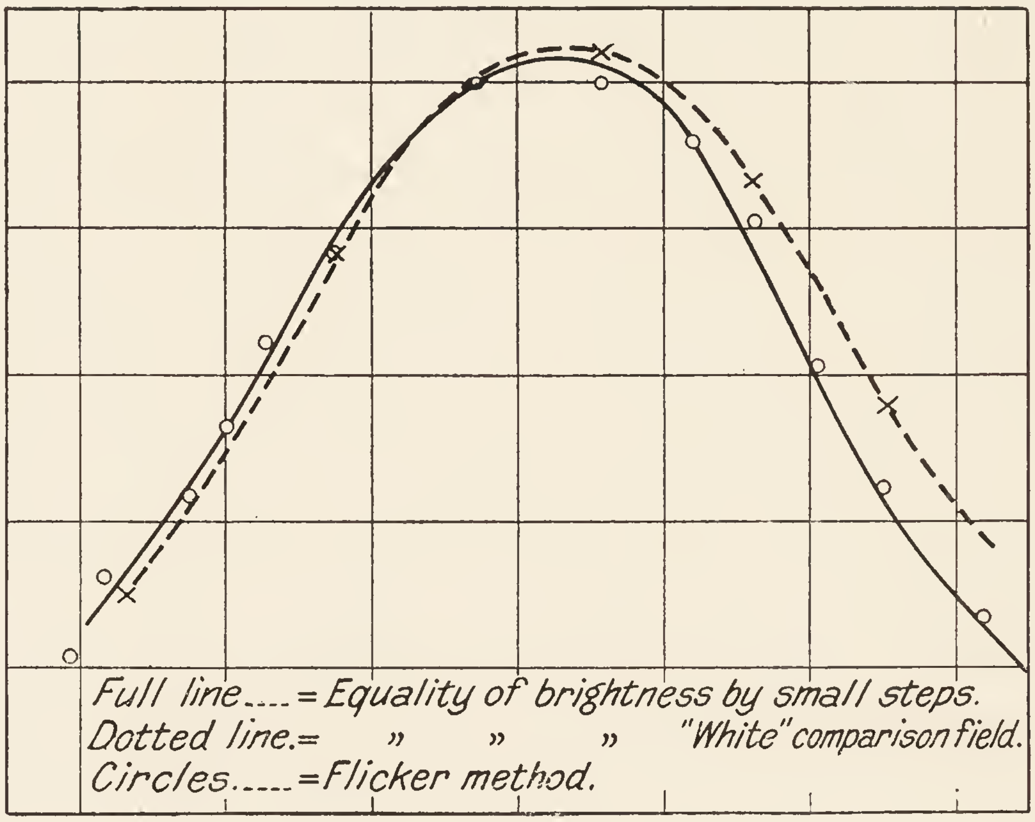

FIG. 12.—Luminosity Curves.

of the resultant white. His data with reference to light passing through three slits placed in the red, green, and violet regions of the spectrum respectively are as follows :—

R	203
(R+G)	242
G	38·5
(G+V)	45
V	8·5
(V+R)	214
(R+G+V)	250

From these we have by summation :

R+G+V	250
(R+G)+V	250·5
(V+R)+G	252·5
(G+V)+R	248
(R+G+V)	250

The mean is found by the method of least squares to be 250·25,

the variations from it not exceeding the possible experimental errors.

Abney's conclusion is a most important experimental result obtained without reference to theory. Yet, as we shall see immediately, the Young-Helmholtz theory is in direct consonance therewith, to the sufficiency of which it therefore affords evidence having the value of the verification of a prediction.

60. **The Visual Sensations. The Resultant Sensation. Surfaces of Constant Luminosity.**—If we use in the first place the unmodified form of Fechner's law, taking x, y, z to represent the external intensities of, say, red, green and blue lights, which produce the sensations ξ, η, ζ respectively, we have

$$\frac{dx}{x}=d\xi,\ \frac{dy}{y}=d\eta,\ \frac{dz}{z}=d\zeta.$$

And Helmholtz used these expressions, as also their extended form, to deduce laws of colour variation from the known laws of intensity variation (§§ 46, 47, 77). Conversely, the laws of combined sensations, the fundamental sensations being regarded as independent, are expressible in terms of the fundamental external stimuli.

If we write the preceding laws in the form

$$\xi=\log\frac{x}{x_0},\ \eta=\log\frac{y}{y_0},\ \zeta=\log\frac{z}{z_0},$$

x_0, y_0, z_0 are the threshold values of the intensities provided that Fechner's law is valid near the thresholds.

Each compound sensation may either be described in terms of its fundamental trichromatic components, or it may be characterized in terms of its apparent brightness, whiteness, and colour tone. We may attempt to adjust two coloured lights to equality in respect of each of the three characteristics independently. In each case a different function is dealt with; and the remark of Helmholtz, quoted above, shows how difficult it is to be sure that the simultaneous presence of one characteristic in unequal amounts does not influence the estimate of equality with regard to another characteristic. Unlike the ear, which can analyze a compound musical note

into its fundamental constituents, the eye is unable to analyze a compound colour into its fundamental constituents. But it can analyze a compound colour sensation in respect to its contents in brightness, whiteness or impurity, and colour tone. These features may therefore be regarded as constituent *visual* sensations.

In addition to the question of the form of the functions which express these visual sensations in terms of the component fundamental sensations ξ, η, ζ, there arises the question of the form of the function of ξ, η, ζ which expresses the *resultant* sensation due to the simultaneous excitation of these fundamentals.

In so far as the above simple expressions for ξ, η, and ζ correspond to observation, any component sensation is doubled if the stimulus is squared, trebled if the stimulus is cubed, and so on; and in each component nothing but the intensity or brightness is altered. Thus if we have $\xi_1 = \log(x_1/x_0)$, $\xi_2 = \log(x_2/x_0)$, we have

$$\xi_1 + \xi_2 = \log \frac{x_1 x_2}{x_0^2}.$$

That is to say, brightnesses are added by multiplying together the component stimuli. If this law held in relation to superposition of brightnesses of the independent fundamental stimuli, we would have

$$\xi_1 + \eta_1 + \zeta_1 = \log \frac{x_1 y_1 z_1}{x_0 y_0 z_0},$$

and surfaces of uniform brightness in the colour pyramid would be given by

$$xyz = c,$$

where c is a constant. See §§ 63, 79.

61. **Luminosity Curves of the Spectrum. Sensation Curves.**—The curves of luminosity throughout the spectrum afford a further important confirmation of the results of the trichromatic theory.

Any colour equation such as

$$c\mathrm{C} = r\mathrm{R} + g\mathrm{G} + b\mathrm{B}$$

makes no assertion regarding the composition of the compound light C. It does not assert that C contains r, g and

b units respectively of particular red, green and blue lights whose unit amounts are denoted by R, G, and B. Its assertion is that c units of the compound light produce the same resultant sensation as the mixture of the r, g and b units produce. Part of that sensation is associated with colour and part with intensity.

In strictness the equation should be written in the form

$$f(c\mathrm{C})=f(r\mathrm{R}+g\mathrm{G}+b\mathrm{B}),$$

where $f(c\mathrm{C})$ means the magnitude of the sensation due to the stimulus provided by an amount c of the light whose unit and character are denoted by C. The form of the function is given by the psychophysical law expressing sensation in terms of stimulus. In so far as the simplest form of Fechner's law is correct its form is that of the logarithm. For in that case we have, if ξ be the magnitude of the sensation corresponding to the stimulus of type R,

$$d\xi=\frac{dr}{r}$$

$$\xi=\log r+\log\mathrm{R}=\log(r\mathrm{R}).$$

The meaning of the constant of integration R is given by putting $r=1$. Thus $\log\mathrm{R}$ is the value of the sensation evolved by unit stimulus, i.e., by unit amount of the external light. The reciprocal of R is the threshold value of the corresponding fundamental stimulus (§ 60). The value of R is arbitrary, for we can give to unit stimulus any value we please; and similarly in the case of G and B.

The curve of luminosity throughout the spectrum, Fig. 14, is, as shown in § 67, determinable directly by comparison with white light. Since the proportions, r, g, b of three suitably chosen constituent lights which are present in spectrum light of any given wavelength, are also determinable directly, in terms of luminosity, by comparison with white in the same manner, the luminosity curves for these components can be similarly found. They are curves of fundamental luminosity sensation.

When the three curves are plotted with the values of r, g, b as ordinates, instead of with equivalent white luminosities as ordinates, it is customary to call them the colour sensation

curves. Both sets can be independently plotted from experimental data. Both are really curves of stimulations producing sensations. There is no usable direct scale for the measurement of sensation apart from the least perceptible differences (§ 29). But the Young-Helmholtz theory enables us to plot true sensation curves by using the logarithms as above.

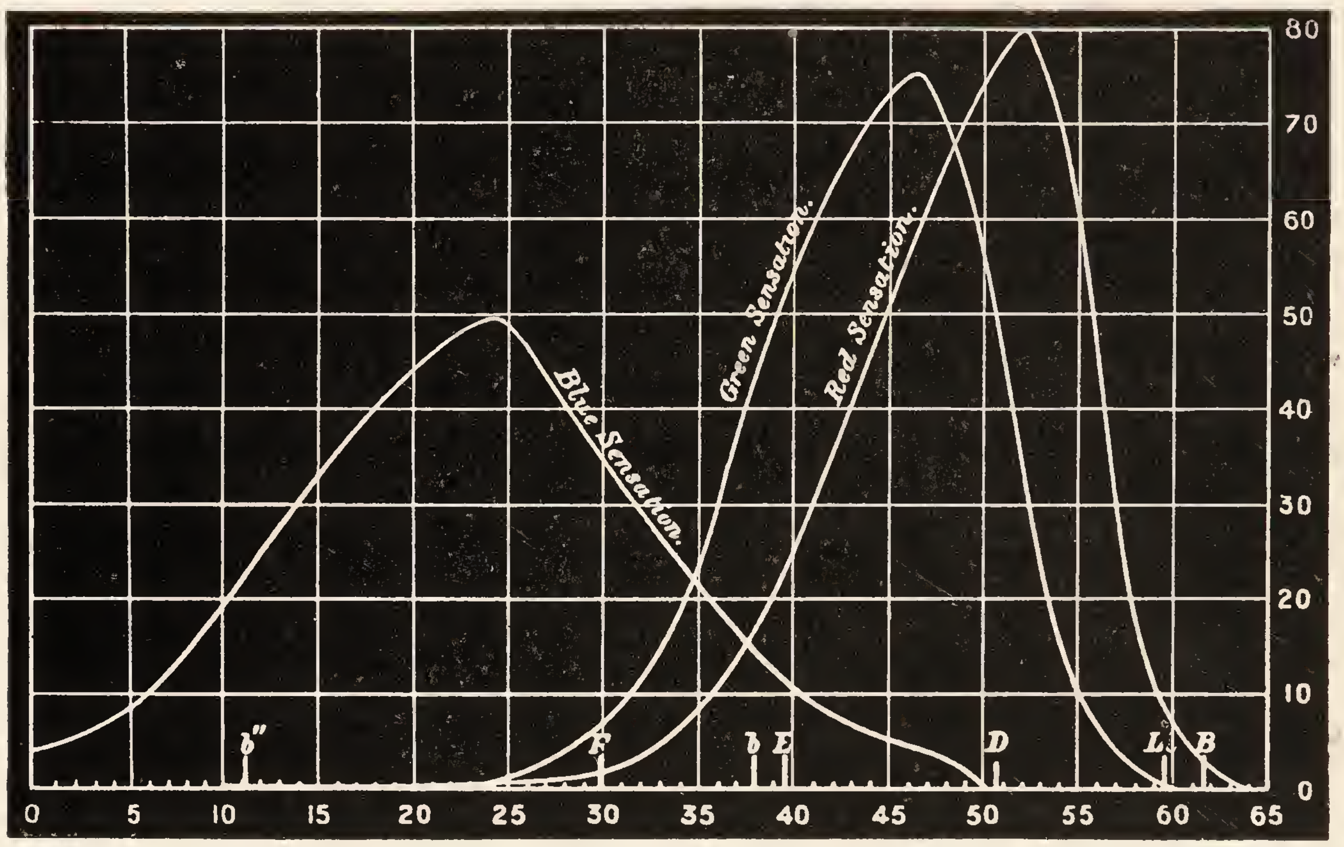

FIG. 13.—Sensation Curves (Equal Area).

62. **Luminosity Curves in Anomalous Trichromasy.**—In Chapter V we have discussed geometrically the possibilities of change of the fundamentals and the peculiarities of vision which may arise therefrom. The analytical treatment, though not appealing so readily to the eye, is even simpler. Let a colour which, as seen by the normal eye, is represented as

$$rR+gG+bB$$

be changed, in abnormal vision, in accordance with the substitutions

$$R=a_1R+b_1G+c_1B,$$
$$G=a_2R+b_2G+c_2B,$$
$$B=a_3R+b_3G+c_3B,$$

originated, it may be, through changes in retinal nerve endings, or through cross leakage in conducting circuits or at

the brain centres. The altered colour is, if the brain centres remain in normal efficiency,

$$(a_1r+a_2g+a_3b)\mathrm{R}+(b_1r+b_2g+b_3b)\mathrm{G}+(c_1r+c_2g+c_3b)\mathrm{B}.$$

In the colour diagram no more than two at most of the perpendiculars on the sides from any point in the plane can be negative. Therefore one at least of the three constants with a given suffix must be positive. And, since no quantity within brackets in the altered colour can be negative in the perception of the normal eye, one a, one b, and one c, at least must be positive. Any light having values of r, g, b in the normal perception, which, with the given values of the quantities a, b, c, makes any quantity within a bracket negative is incapable, so far as that component is concerned, of perception by the normal eye if we presume, as just noted, that the final perceptive mechanism in the brain is intact and normal, its mode of excitation alone being abnormal.

The three conditions

$$a_1r+a_2g+a_3b \quad 0,$$
$$b_1r+b_2g+b_3b=0,$$
$$c_1r+c_2g+c_3b=0,$$

together with the three

$$r=0, \quad g=0, \quad b=0,$$

provide the limits within which any colour perceptible to the abnormal eye must lie. Each of these six conditions is the equation of a plane, which, when combined with the equation of the plane of the diagram, say,

$$r+g+b=1,$$

gives a line in the diagram which forms one of the bounds of the colour perceptive area. Since there are not more than six limiting conditions, there cannot be a more complicated figure than a hexagon. If one of the bounding lines lies more remote in the diagram than the intersection of the two neighbouring bounding lines, the hexagon is replaced by a pentagon, and so on in accordance with the geometrical scheme sketched in Chapter V.

The various types of cases are great in number. We shall only discuss here further the special cases in which there is

merely lessening of perception without alteration of the fundamentals.

In the most general case of this kind the normal colour above becomes

$$prR + qgG + sbB,$$

and we may, in the manner sketched in §§ 59, 61, determine the luminosity curve of the spectrum as seen by an eye possessing this mode of perception, and also the component luminosity curves. The normal eye compares the luminosity of the colour $rR + gG + bB$ with that of the combined white of the spectrum

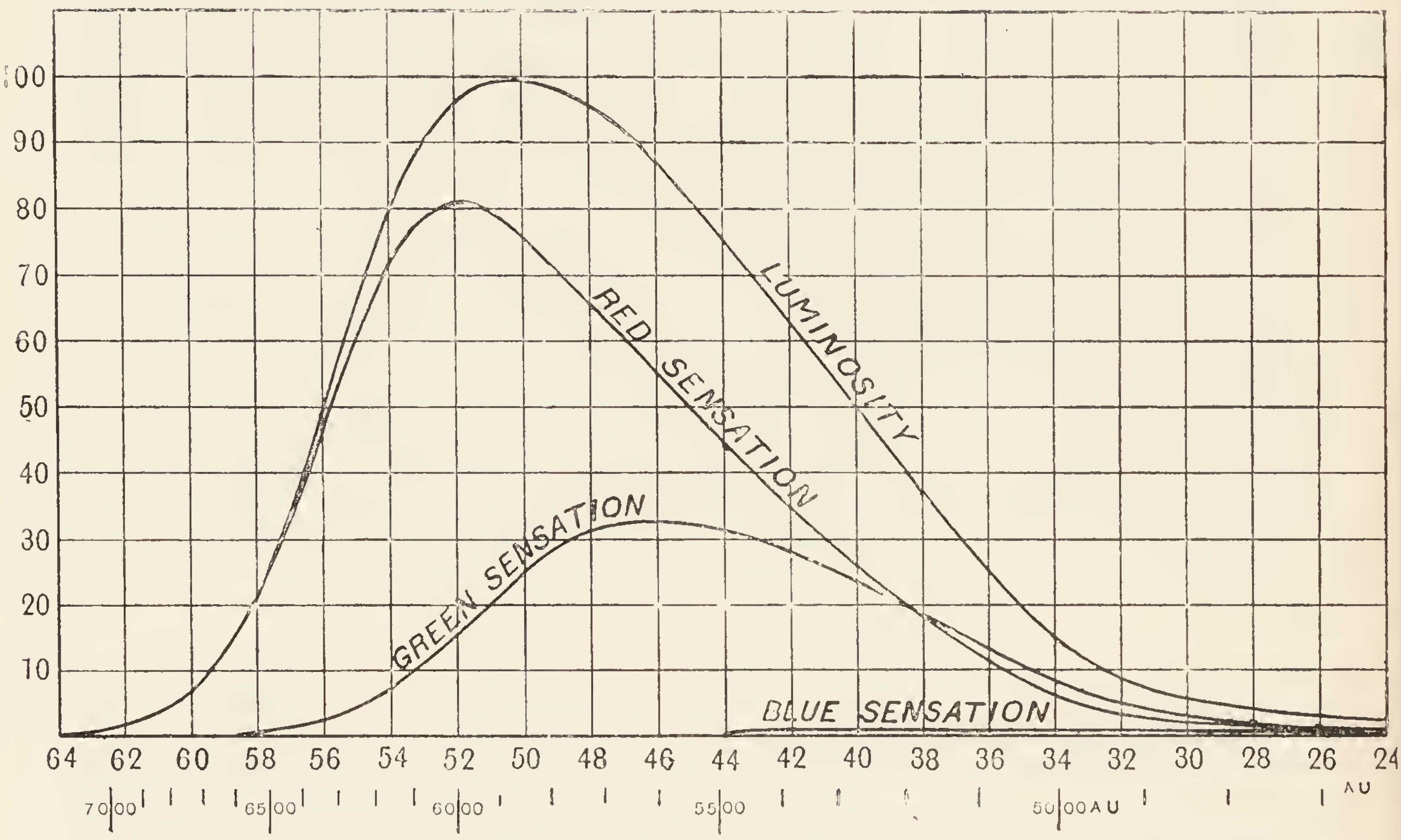

Fig. 14.—Sensation Curves (by Luminosity).

as seen by it ; while the abnormal eye compares its perception $prR + qgG + sbB$ with its own perception of white. If the component luminosity curves are plotted to a scale of wavelength (Fig. 14) the product of the ordinate, say r, into $d\lambda$ gives the luminosity of the portion of the spectrum included in that range of wavelength in so far as the red sensation is concerned. And the total luminosity of the red sensation in the spectrum is represented by the total sum of these quantities over the whole range of visible wavelengths. It is therefore given by the total area of the curve for the red sensation, A_r say, and similarly in the case of the other sensations. Thus

the ratio of the luminosity of the range $d\lambda$ to that of the whole spectrum, being the ratio of the sum of the ordinates of the three sensation curves multiplied by $d\lambda$ to the sum of the areas of the sensation curves, that is, to the area of the resultant luminosity curve, is

$$\frac{(r+g+b)d\lambda}{A_r+A_g+A_a}$$

in the case of the normal eye; while, in the case of the abnormal eye, it is

$$\frac{(pr+qg+sb)d\lambda}{pA_r+qA_g+sA_b}.$$

Now if we can find any wavelengths for which the conditions

$$\frac{r}{A_r}=\frac{g}{A_g}=\frac{b}{A_b}=k$$

hold, where k is a constant, the values of the normal and abnormal ratios become equal quite independently of the values of the reduction factors p, q, s. This means that the spectrum luminosity curves of all trichromatic observers, whose fundamental perceptions differ from the normal perceptions by constant multipliers only, have a common ordinate at the wavelength for which the normal fundamental sensations bear one constant ratio to the areas of the corresponding sensation curves.

Anomalies of vision of this nature occurring in trichromatic vision may be designated Magnitude Anomalies.

When the sensation curves are plotted thus in terms of luminosity, the ordinates and area in the case of the blue sensation curve are very small in comparison with those of the red and green sensation curves. Neglecting the blue sensation curve, and using the red and green sensation curves only, Watson has made an elaborate investigation of many cases of anomalous trichromasy belonging to types which, in Abney's view, exhibited partial red blindness or partial green blindness. The observations verified Abney's conclusion and the accuracy of this result of the trichromatic theory. Watson's theoretical curves, calculated for different amounts of reduction in the red and green sensations, are shown in Fig. 15.

In such observations the effects of absorption due to excess or defect of pigmentation in the eye with respect to the normal amount have to be carefully taken into account. When there is excess of absorption in the green region of the spectrum, the comparison white is more affected than the longer wavelength region, and so the curve of luminosity lies above the normal curve in the neighbourhood of, and on the red side of, the wavelength where the common ordinate should be found; it lies below the normal curve at the shorter wavelengths where the absorption is manifested.

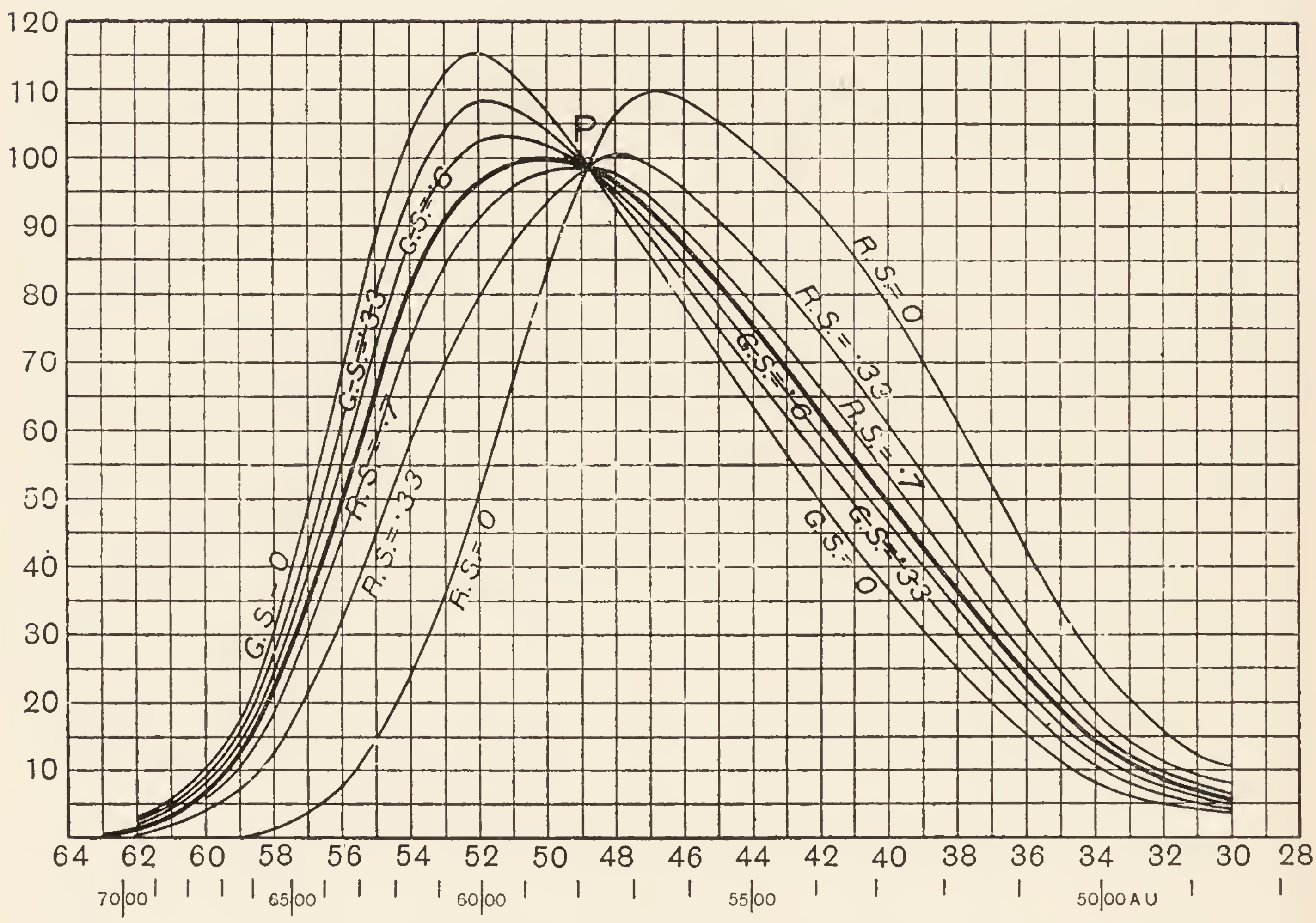

FIG. 15.—Theoretical Luminosity Curves.

Apart from the effect of pigmentation, the method given above can be used in the discrimination of partial deficiency in any or all of the three fundamental sensations. Instead of a single curve corresponding to a definite reduction in one sensation, a few would have to be drawn corresponding to definite reduction values for another third sensation; and, corresponding to each of these, a few more would have to be drawn corresponding to different reduction values for the third sensation. More than three hundred in all would be requisite

to give as complete representation for the general case of anomalous trichromasy, as Watson's single diagram gives with regard to the two sensations when the third is neglected: forty-nine if the luminosity were constant.

On the other hand, it must be remembered that his diagram, because of neglect of the blue sensation, cannot be extended into the blue region of the spectrum. But a single diagram like his, modified so as to include the full blue sensation, could easily be made.

63. **The Sensation Value. The Law of Equivalent Sensation Values.**—The simplest function which can be taken as representative of a sensation in terms of the corresponding stimulation being the logarithm of the ratio of the stimulation to its threshold value, we may put

$$S_r = \log \frac{r}{r_0},$$

where S_r is the sensation induced by the red stimulation r of which r_0 is the threshold value, and so on. The ratio of r to r_0 may be termed the Sensation Value of the light. If some other light, w, be selected as a standard of reference, and if w_0 be its threshold value, while w_r is the amount of it which gives rise to a sensation equal to S_r, the equivalence is expressed by the equation

$$\log \frac{r}{r_0} = \log \frac{w_r}{w_0}$$

or

$$\frac{r}{r_0} = \frac{w_r}{w_0}.$$

Similarly

$$\frac{g}{g_0} = \frac{w_g}{w_0}$$

and

$$\frac{b}{b_0} = \frac{w_b}{w_0}.$$

If w_c be the standard stimulation equivalent to the compound light C whose components are r, g, b, we get

$$w_c = w_0 \frac{f(r, g, b)}{c_0},$$

where $c=f(r, g, b)$ and c_0 is the threshold value of the compound light. It is unnecessary at present to discuss the form of the function f. We have also

$$w_r+w_g+w_b=w_0\left(\frac{r}{r_0}+\frac{g}{g_0}+\frac{b}{b_0}\right).$$

But if, along with $c=f(r, g, b)$, we substitute in the colour equation representing the Newtonian law of colour mixture

$$c\mathrm{C}=r\mathrm{R}+g\mathrm{G}+b\mathrm{B},$$

the values (§ 61)

$$\mathrm{R}=\frac{1}{r_0},\ \mathrm{G}=\frac{1}{g_0},\ \mathrm{B}=\frac{1}{b_0},$$

which enable us to cut out any term in which the stimulus falls to or below its threshold value, we have

$$\frac{f(r, g, b)}{c_0}=\frac{r}{r_0}+\frac{g}{g_0}+\frac{b}{b_0}.$$

Therefore

$$w_c=w_r+w_g+w_b.$$

This is the law of addition of equivalent sensation values; and it is a direct consequence of Fechner's psychophysical law together with Newton's law of colour mixture. The form of f will depend upon the particular aspect of the sensation, e.g., brightness or colour strength, which is under consideration (§§ 67, 73, 76).

64. **White Light. Invariance of the Law with respect to the Fundamentals.**—There is no necessary limitation on the choice of the standard light w, provided only that it can affect all three fundamental sensations. The only symmetrical choice is that of the light which calls forth all three fundamental sensations in equal amount. It is a matter of the greatest significance that the theory suggests the existence of a peculiar light possessing an aspect of symmetry with regard to the three necessarily postulated fundamentals and to the totality of colours compounded of them. In exhibiting no bias towards any colour, it possesses the characteristic of the light which we call white. No consequence of the theory could more fully illustrate the prevision of its great founders in basing it upon

the simple tripleness of colour perception, which no theory can avoid, and from which all the other properties flow. Such results are at once the test and the proclamation of the presence of the intuition of genius, which is ever in touch with the essentialities of nature.

The light which is neutral towards all colour should have properties which are independent of any arbitrary choice of particular fundamentals. That this is so may be seen at once from the above equations. For each term is a ratio of two quantities of the same kind, so that any change in the value or composition of the unit is devoid of influence upon the magnitude of the term. Thus, for the same colour referred to different fundamentals, we have the universal conditions

$$\frac{f(r', g', b')}{c'_0} = \frac{r'}{r'_0} + \frac{g'}{g'_0} + \frac{b'}{b'_0},$$

$$w'_c = w'_r + w'_g + w'_b.$$

In the development of a triplex organ of vision under the influence of external light, the most universal perfection of vision would be associated with entire triple symmetry towards that external light. This leads directly to the condition (§ 17) that white light is that light to which the eye is most accustomed. In anomalous trichromasy the symmetry is incomplete; yet the additive law with regard to sensation values remains.

65. **Application of the Theory of Pure Strains.**—The three equations (§ 63) connecting w_r, w_g, w_b with r, g, b show that a pyramid of sensation values can be formed from the colour pyramid, i.e., the pyramid of colour stimuli, by applying to it a definite pure strain, whose axes are the co-ordinate axes, and whose principal semiaxes are the ratios of the threshold value, w_0, to the threshold values r_0, g_0, b_0 of the three fundamental lights. When the threshold values are known the equations give complete information regarding the whites equivalent in sensation value to the fundamental colour components of any given light. But the theory of pure strains at once gives us elegant geometrical relations amongst the colour components and the equivalent white components.

Thus a point which, in the colour pyramid, lies on the sphere

$$r^2+g^2+b^2=1,$$

lies, in the equivalent white pyramid, on the ellipsoid

$$\frac{b^2{}_0}{w^2{}_0}.\ b^2{}_r+\frac{g^2{}_0}{w^2{}_0}.w^2{}_g+\frac{b^2{}_0}{w^2{}_0}.w^2{}_b=1,$$

whose principal semiaxes have the values stated above. The projections on the co-ordinate axes, of the line joining that point to the origin, are the values of w_r, w_g, w_b; and the direction of the normal to the ellipsoid at the point is the direction of that line which, in the colour pyramid, indicates the colour whose components are r, g, b.

Conversely, a point which, in the white pyramid lies on the sphere

$$w^2{}_r+w^2{}_g+w^2{}_b=1,$$

lies, in the colour pyramid, on the ellipsoid

$$\frac{w^2{}_0}{r^2{}_0}.r^2+\frac{w^2{}_0}{g^2{}_0}.g^2+\frac{w^2{}_0}{b^2{}_0}.b^2=1,$$

whose principal semiaxes are the reciprocals of the former semiaxes. The projections, on the co-ordinate axes, of the line joining that point to the origin, are the values of r, g, b; and the direction of the normal to the ellipsoid at the point is the direction of that line which, in the white pyramid, exhibits the components by projection on the axes.

But these results, though interesting, contain no essential information beyond that contained in the three equations.

66. **Persistency and Persistivity.**—The equation

$$w_c=w_0\frac{c}{c_0}$$

is, in terms of Fechner's law, the expression of the experimental condition that the sensation values of the compound light and its equivalent white light are equal. The additional information which Fechner's law gives is that all lights have equal sensation values when their amounts are the same multiples of their threshold values. The yet further information given by Newton's law as restricted by the Young-Helmholtz theory, and expressed by the equation

$$\frac{c}{c_0}=\frac{r}{r_0}+\frac{g}{g_0}+\frac{b}{b_0},$$

is that of the independent summation of the sensation values of the constituent lights. The law

$$w_c = w_r + w_g + w_b$$

makes the similar statement regarding the equivalent white lights of corresponding total and component sensation values.

The fraction c/c_0, expressing the sensation value of the light c, is the number of " just visible " units which the light contains. It therefore measures the range throughout which the light may be reduced in strength without becoming invisible. It

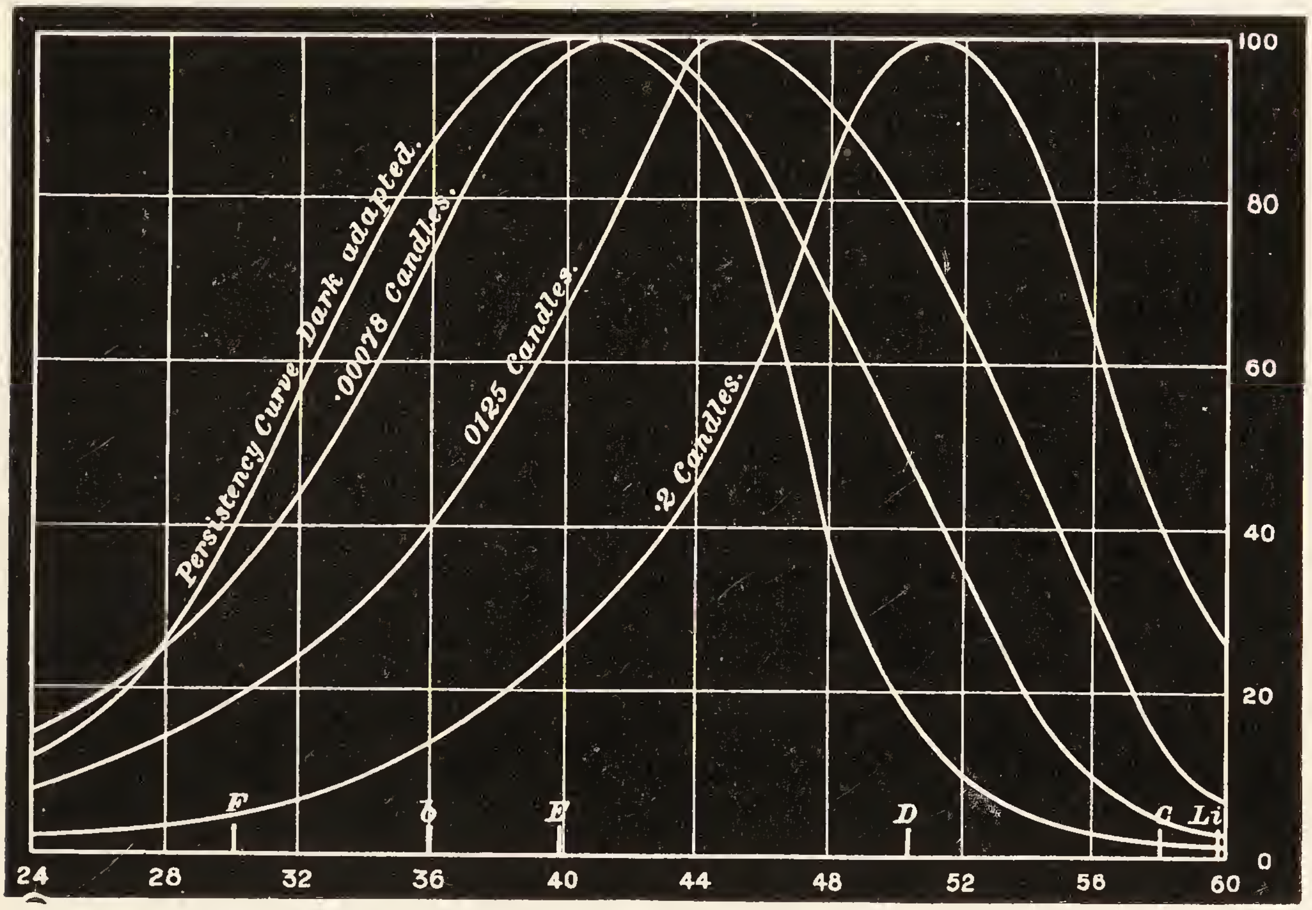

FIG. 16.—Persistency Curves.

may therefore be alternatively termed the Persistency of that particular light. A curve representing its value throughout the spectrum for each wavelength is the Persistency Curve of the spectrum. The persistency is taken with regard to the actual stimulus or intensity of the light as it exists at each wavelength throughout the spectrum. By " intensity " here is not meant necessarily the physical intensity, but its stimulative value. This does not necessarily bear the same proportion to the physical intensity at different wavelengths. A curve of persistency in the spectrum is a luminosity curve

(§ 61). Some of the former type, as determined by Abney, are shown in Fig. 16, and two of the latter type, given by Abney and Festing, are shown in Fig. 17.

The fraction $1/c_0$, expressing the number of "just visible" units contained in light of unit intensity of a given wavelength, measures the intrinsic or specific persistency of that kind of light. It may therefore be termed the Persistivity of that light. A curve of spectrum persistivities is the curve reciprocal to a curve of threshold values.

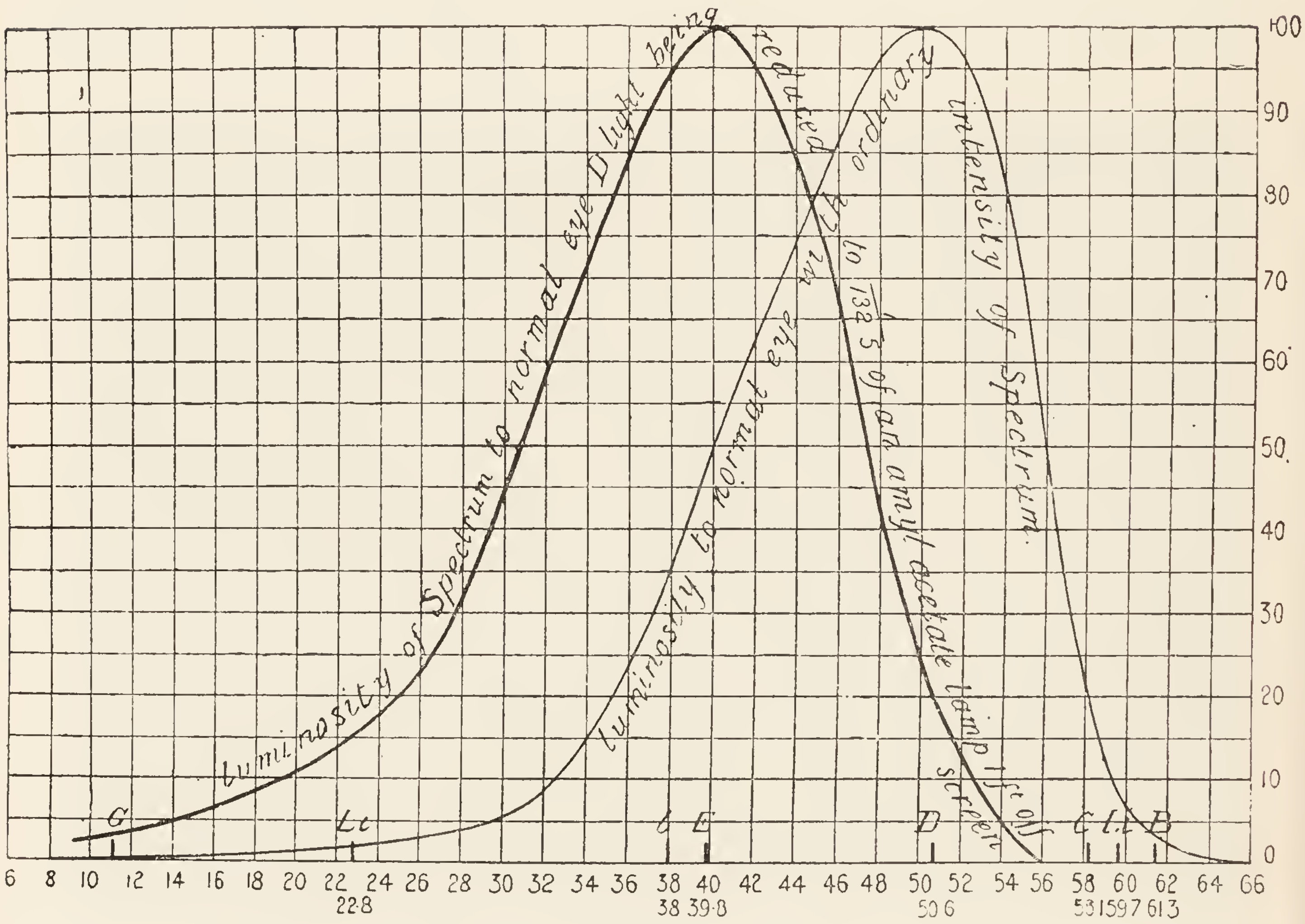

Fig. 17.—Curves of Strong and Weak Luminosity.

The right-hand side of the equation

$$\frac{f(r, g, b)}{c_0} = \frac{r}{r_0} + \frac{g}{g_0} + \frac{b}{b_0},$$

if it refers to a spectrum light, gives the persistency of the light in terms of its fundamental component persistencies which are summed on that side. The equation is essentially an equation for the determination of the resultant persistivity or threshold value when the fundamental persistencies and the function f are known.

The equation, as we have seen, is derived from the usual equation

$$cC = rR + gG + bB$$

through alteration of the numerical values of the co-efficients in accordance with the considerations that the magnitude of the sensation produced by any given light vanishes at a definite threshold value of the intensity, and that Fechner's law gives the bond between the sensation and the stimulus. The latter equation, when applied to the general resultant sensation, is really a *vector* equation, in which C, R, G, and B represent unit vectors, the three last mutually rectangular, whose tensors are c, r, g, and b respectively. This fact was well known to Helmholtz. He says that "Newton's centre of gravity construction, which we have hitherto used, is, as the analysis into fundamental colours distinctly shows, only an intuitive presentation of a much more generally occurring form of the interaction of qualitatively different quantities, to which, as Grassmann has likewise shown in very general manner, belong the essential characteristics of addition. The first weighty example of an additive connection of non-homogeneous quantities was given by Gauss, in that he attached geometrical signification to the complex quantities of algebra. Extended in another form, the same reappears in the system of quaternions developed by Hamilton." Thus from the vector aspect, directional signification has to be attached to the symbols $1/r_0$, etc., in the former equation. The magnitude of the units alone is changed.

The unit of the red stimulus in the last equation above is r_0 times greater than the unit of the red stimulus in the immediately preceding equation.

67. **Luminosity. Abney's Law.**—It is desirable to consider yet more fully the nature of the transformation from the usual form of the colour equation to the form involving the introduction of the threshold values into the several terms. Let the observed numerical values of the compound colour and its fundamental constituents be c, r, g, and b respectively. These are in themselves entirely arbitrary. We have to find the restriction which has to be imposed in order that the presumption of the correspondence of white light to equality

of the fundamental sensations shall be upheld. To effect the transformation we write

$$c' = k\frac{c}{c_0},\ r' = k\frac{r}{r_0},\ g' = k\frac{g}{g_0},\ b' = k\frac{b}{b_0},$$

which give

$$\frac{c}{c_0} = \frac{r}{r_0} + \frac{g}{g_0} + \frac{b}{b_0}.$$

Hence the equality of the three fundamental sensations in the case of white light necessitates the condition that the relations

$$r' = g' = b'$$

shall obtain for white light. It is therefore necessary that, in the experimental determination of the colour equations, the proportions in which the fundamental lights are present in white light shall be found, and that units bearing these same proportions to each other respectively shall then be employed, so that the areas of the sensation curves are made equal.

That being done, we have

$$r' = k\frac{w_r}{w_0},\ g' = k\frac{w_g}{w_0},\ v' = k\frac{w_b}{w_0},$$

so that the values of the component fundamental lights or subjective stimuli are proportional to the values of the equivalent white components.

If we consider now the form of the function f which will be applicable when the sensation of luminosity is concerned, we may tentatively adopt the simplest postulate that the constituents of the sensation value of the luminosity are proportional to the components of the fundamental objective stimuli. Thus we have

$$l_r = pr = \frac{p}{k} r_0 r' \ :\ l_g = \frac{p}{k} g_0 g' \ :\ l_b = \frac{p}{k} b_0 b',$$

or

$$l_r = \frac{p}{w_0} . r_0 w_r \ :\ l_g = \frac{p}{w_0} . g_0 w_g \ :\ l_b = \frac{p}{w_0} . b_0 w_b .$$

Thus the luminosity components are proportional to the components of the external stimuli or the equivalent whites, and to the corresponding threshold values, conjointly.

Now as the fundamentals are alterable by linear substitutions without alteration of the colour relations, we put

$$l_c = \frac{p}{w_0} . c_0 w_c = pc = pf(r,\ g,\ b).$$

The form of the function f can be determined by an experimental evaluation of c_0 in terms of r, g, and b. Otherwise, the form of f may be postulated and the resulting expression for c_0 may be compared with the observed threshold values.

If we presume

$$l_c = l_r + l_g + l_b$$

we get

$$c = r + g + b,$$

or conversely. In either case the law is the simplest which could be presumed. The former expression, being invariant in form, although the fundamentals be changed by linear substitutions, asserts that the luminosity of a light is the sum of the luminosities of its components. This is Abney's law of the addition of luminosities (§ 59). The experimental establishment of that law is a sufficient basis for the adoption of that expression. The accuracy of the deduction contained in the second expression is obvious at once from the relations which show that the two expressions are identical term by term, apart from a common multiplier which may be made unity.

The diagrammatic representation of the expression for f is given in the upper curve of Fig. 14 (p. 104).

From the relation

$$\frac{r+g+b}{c_0} = \frac{r}{r_0} + \frac{g}{g_0} + \frac{b}{b_0}$$

we now find at once

$$\frac{k}{c_0} = \frac{r' + g' + b'}{r + g + b}.$$

Just as the denominator is the sum of ordinates in Fig. 14, so the numerator is the sum of ordinates in Fig. 13. Therefore, from the two diagrams, we can calculate the persistivities, or the threshold values, throughout the spectrum, for k is made unity in the construction of the diagrams. At the scale

numbers given in the upper row immediately following, we find the values of the persistivity given in the under row—

60	55	50	45	40	35	30	25
1	1·1	1·3	1·55	1·82	2·83	7·72	29·0

which indicate a continuous decrease throughout the spectrum from short wavelength to long wavelength ; very rapid towards the blue region, very slow towards the red. This is in entire accordance with observation.

If we plot the persistency $r'+g'+v'$ against scale number, we get a concordant curve throughout the range specified above. A deviation appears at still shorter wavelengths, seemingly due to uncertainty in the blue sensation curve.

68. **Stimulation and Sensation.**—The wonderfully penetrative discussion of the general relation between cause and effect in colour perception given by Helmholtz (§ 27) should be read again by any student of the subject, in the light of the conclusions just reached. Helmholtz's quantities F, x, y, z correspond to the quantities c', r', g', b' in the preceding section. The quantities r/r_0, g/g_0, b/b_0, which are the " sensation values " in the terminology here used, may be regarded as his ϕ, ψ, χ, " which together define completely the sensation of the eye." His r, g, v correspond to kr/r_0, kg/g_0, kb/b_0.

The intuitional fulness of vision of his prediction that the sensations attaining to consciousness shall themselves correspond to simple and very directly related functions of the excitations is significant. It was based solely upon the close correspondence of colour names, which in ordinary speech arise as the immediate expression of observed differences and similarities, to the features of the arrangement of colours in the trichromatic colour pyramid. The sensation values are related, as we see, approximately at least in direct proportion to the stimuli ; and the sensation itself varies in geometrical progression as the stimulus varies in arithmetical progression.

Regarded as a first approximation only, the results of the theory follow the facts very closely throughout a wide range of observation. This first basing is upon the unmodified form of Fechner's law. And we have seen how Fechner's own modification of the law, necessarily introduced in order to include the widened basis of fact made evident, as Helmholtz

showed, in connection with the self light of the eye, pushes the efficient range of correspondence far in the direction of feeble illumination ; and how Helmholtz's introduction of the dazzle factor extends it in the direction of strong stimuli. We have seen also how Fechner's law compels recognition of the existence of threshold stimuli. These are not proppings of a weak structure. They are the gradual, purposeful, unavoidable extension of cantilever arms for the reception of the completing girder.

Ties and struts, latent in the design, grow in with the growth of the embodiment ; and the embellishments, great and small, give it the stamp of that beauty in unity which belongs to the imaginings that are in touch with nature. In the first approximations, the quantities which specify the self light and the threshold stimuli are regarded as constant. In the second approximations they must be taken as variable. And beyond these considerations lies the question of the possibility of deviation from Newton's law : but it, like his law of gravitation, seems to be very immune. Apparent deviations, like those manifest in very weak or in very strong illumination—such, for example, as Purkinje's phenomenon, the shift of the position of maximum brightness in the spectrum with varying intensity of the light, which is very evident in the curves of Fig. 16, p. 111, seem to throw light upon the nature of the mechanism of colour vision rather than upon the law itself.

CHAPTER IX

DIFFERENTIAL SENSITIVITY AND THE ABSOLUTE FUNDAMENTALS

69. **The Sensation Curves and their Determination.**— To the great central peak of knowledge regarding colour one and only one line of approach is open. Information may come through anatomy or through physiology giving verification of precedent, or a basis for subsequent, presumptions regarding the nature of vision. Such information, if complete, would give direct access to the scalable ridge. Being incomplete, it leaves the line of the path to be settled by inference. Just as physical science. founding upon the simple Newtonian law of gravitational attraction, and taking for its observed data the curves of planetary perturbations, gave to astronomy the completion of the major planetary system ; so here, founding upon the simple Newtonian law of colour mixture, and taking for its data the observed curves of the colour sensations, physical science alone is in the position of ability to point to an adequate solution of the optical problem.

The simplicities of the fundamental laws cannot be ignored. They are of the essence of the problem.

The determination of the colour sensation curves for the spectrum in the case of trichromatic vision is a comparatively simple matter, though complications may enter in connection with the type of apparatus employed. In the case of dichromasy the determination is still more easy.

Since it is always found that the colour obtained by mixture of two spectrum lights is whiter than the similarly coloured region of the spectrum, it follows that, if we associate with the three corners of an equilateral triangle definite amounts of approximately monochromatic lights from three selected parts of the spectrum situated respectively in the middle region,

and towards the two end regions, of the spectrum, the curve of the spectrum in the resulting colour diagram must circumscribe the triangle so as to lie entirely outside it except at the three corners. It would be convenient, as we have seen, to take, as unit amounts of the three standard lights, amounts which when combined together produce white, and to locate these unit amounts at the corners of the triangle. The centre of the triangle then corresponds to white light.

If then definite proportions of, say, the red and green standards are mixed together so as to match, say, a yellow light of definite wavelength when mixed with the necessary amount of white light, the position of that yellow spectrum light in the colour diagram is rigidly fixed. The ratio of the amounts of red and green gives, in accordance with Newton's centre of inertia construction, the position of the point in which the line drawn from the centre towards the yellow point of the spectrum cuts the red-green side of the triangle. And, similarly, the ratio of the amount of the yellow spectrum light to the amount of the white which has to be added to it in order to complete the match, determines the position, on the white-yellow line, of the definite yellow spectrum light under consideration. In this way the whole spectrum curve can be drawn, each point necessarily corresponding to the condition that the three perpendiculars drawn from it to the sides of the triangle, are proportional to the amounts of the three standards employed. As the exact amount of any one of them, say the green, can be measured, the values of the three ordinates of the three component sensation curves are determinable throughout the range of the spectrum.

In a case of dichromasy the mid spectrum standard would be omitted.

The other method of determining the colour sensation curves consists in finding the amounts of white light which are respectively equivalent in sensation value to each of the three standard lights present as components in the compound light which matches some given spectrum colour with white added to it if necessary. It is therefore not at all a different method of analysis from the former. It merely uses a different method of estimation of the magnitudes of the three components, which requires justification, either experimentally by com-

parison of the resulting sensation curves with those found by the other method, or by the theoretical reconciliation which we have found to be furnished (§ 67) by the Young-Helmholtz theory. The experimental justification being adopted, we have, in the coincidence of the theoretical result, a powerful confirmation of the trichromatic view.

70. **The Selection of Fundamental Colours.**—The selections of three spectrum colours in the manner just described has many practical advantages. If the solar spectrum be used, with fundamental wavelengths in regions free from selective absorption in the eye, it gives easily reproducible standard conditions for observations by different eyes. But these obviously cannot be the absolute fundamentals of the Young-Helmholtz theory which admit of no negative stimulations. The triangle of smallest dimensions which could satisfy this condition would be the one most closely surrounding the spectrum curve. König's first selected fundamentals approximated to this condition (Fig. 3, p. 54). There can be no a priori reason for the selection of such fundamentals apart from the presumption that evolution of the colour sense under the action of solar light might be expected to give a spectrum curve corresponding most closely to the fundamental stimuli which had been differentiated. Yet this idea has little more foundation than has that of the ancients who presumed that the paths of the planets must be circular because the circle is the perfect curve. In any case the fact of the possibility (§ 38) of intensifying the apparent purity of spectrum colours shows that the absolute fundamentals and the triangle determined by them must lie outside the spectrum curve. The diagram may be distorted so as to make the small triangle equilateral. It is sufficient to regard it from a point of view which gives it an appearance of equal-sidedness.

We have seen that the simplest possible view of the origin of dichromasy consonant with the trichromatic theory is that of the failure of one of the fundamentals. This suggested to König and Dieterici that it might be possible to choose three fundamentals for normal vision such that failure of one or other out of two of the three might account for the two most usual types of dichromasy in which respectively there is special deficiency in the red and the green regions of the spectrum.

They found that the choice could be so made with fairly good results. The corresponding triangle is shown in equilateral form in Fig. 3, p. 54.

Though this result is strongly confirmatory of the Young-Helmholtz theory, it in no wise proves that these fundamentals are the true absolute set: but it certainly indicates the existence in common in these cases of a structural, or functional relationship which, by a mere linear transformation, can be expressed in that simplest of all possible forms. Dichromasy may theoretically result from any linear transformation which makes the points representing the three derived fundamentals lie in one straight line. And so the particular colour which fails to produce any sensation in a definite dichromatic eye occupies a position, relatively to the corners of the presumed fundamental triangle, which is entirely dependent on the arbitrary choice of the three fundamentals. It would, however, have to be admitted that if, in the case of a triple set of "red," "green," and "blue" fundamental sensations found to be suitable for the characterization of a very large number of cases of closely normal colour vision, the failure of the red sensation on the one hand and of the green on the other closely characterized respectively the dichromatic vision of very large numbers of the two types of dichromatic eyes which mainly occur, considerable probability would be given to the presumption that these three fundamentals were the absolute fundamentals. For this is formally the simplest way in which dichromasy can arise. But the actual manner in which dichromasy arises is unknown. If dichromasy actually arises, not from failure of any one component action but from failure of its independence of another component action, the whole basis for the supposition that the three fundamentals selected as above are the absolute fundamentals vanishes.

In the latter event the failing colour lies either on a side of the colour triangle or quite outside it, inhibition apart (§93).

71. **The Failing Colour in Dichromasy as a Non-absolute Colour.**—The value of a determination of the failing colour lies in the fact that it enables us very easily to specify the series of definite different colours as seen by a normal eye which, to the dichromatic eye, present the appearance only of more or less bright varieties of one definite colour. For, if we

choose any one point in the colour diagram and join it to the point at which the failing colour lies, the law of colour mixture shows that the stimulus at any point on the joining line is due to a fraction of the given definite light together with a fraction of the failing light which does not attain to perception. Thus, if we draw through the failing colour triangle a sheaf of straight lines intersecting the normal colour triangle, each of these lines is a monochromatic line to the dichromatic eye (Fig. 11, p. 93).

Now the failing colour may lie, as already said (see below), anywhere in the plane of the colour triangle except in the interior of that triangle. Therefore, through any point in the interior, a monochromatic line may be drawn in any direction whatsoever, so far as the theoretical conditions are concerned. That is to say, on the Young-Helmholtz scheme, a dichromatic eye could be specified which would be quite unable to discriminate between any two definite but randomly chosen colours. Yet there are critics who imagine that there are special cases of dichromasy which are outwith that scheme.

In particular, we see that a dichromatic neutral point (confusable with white, that is) could theoretically be arranged for at any point of the spectrum, together of course with its complementary point in or without the spectrum.

The colour triangle spoken of here may be the absolute triangle or any one of the infinity of non-absolute triangles.

Let x, y, z be the colour components referred to the three rectangular axes of the colour pyramid, absolute or non-absolute, constructed with reference to normal vision. Let x', y', z' be the corresponding components, linearly related to the former, which hold for an abnormal eye. The linear relations referred to specify completely the positions of the abnormal axes relatively to the normal. We have to determine the condition for dichromasy and the location of the failing colour in any special case. The linear relations themselves specify the nature of the abnormality of vision. Let them be as follows. The abnormal "red" centre is to be affected in part by the stimulus which normally affects that centre and in part also by the stimuli which normally affect the "green" and "blue" centres alone. If l_1, m_1, n_1 are respectively the fractions of the normal "red," "green," and "blue" stimuli

x, y, z which reach the red centre of the abnormal eye and become effective in stimulating it, we have

$$x' = l_1 x + m_1 y + n_1 z.$$

Similarly

$$y' = l_2 x + m_2 y + n_2 z,$$
$$z' = l_3 x + m_3 y + n_3 z.$$

If now some particular normal colour, whose normal components have definite values x_0, y_0, z_0, is absolutely ineffective in stimulating the abnormal eye, each fundamental abnormal stimulation must be independently zero when the light $x_0 + y_0 + z_0$ falls on the abnormal eye.

That is, we must have $x'_0 = 0$, $y'_0 = 0$, $z'_0 = 0$, or

$$l_1 x_0 + m_1 y_0 + n_1 z_0 = 0,$$
$$l_2 x_0 + m_2 y_0 + n_2 z_0 = 0,$$
$$l_3 x_0 + m_3 y_0 + n_3 z_0 = 0.$$

Therefore the relation

$$\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

must be satisfied by the nine fractions above specified. Whenever it is satisfied, any two of the preceding equations in x_0, y_0, z_0 determine the position of the failing colour in the diagram.

But this condition is precisely the condition that the points which represent, in the x, y, z colour diagram, the abnormal fundamentals x', y', and z' should lie in one straight line. But this again is precisely the condition, already discussed, that the abnormal vision should be dichromatic. So, in any case of dichromasy, there is some one normal colour which fails.

The fractions l_1, etc., of the stimuli being positive, the equation in x_0, y_0, z_0 show that one at least of these three quantities must be negative in general. Therefore, without inhibition, § 93, the failing colour cannot lie inside the normal colour triangle. Therefore all lights which affect the normal eye also affect the dichromatic eye, though in general they originate a different sensation from that which they arouse in the normal eye.

In particular, if $l_1+m_1+n_1=l_2+m_2+n_2=l_3+m_3+n_3$ we get $x'=y'=z'$ when $x=y=z$. That is to say, the dichromatic sensation of white may coincide with the normal sensation of white.

The theory provides also for an infinity of cases, should any of them exist, in which white light originates a coloured sensation, or a coloured light excites a white sensation, in the dichromatic eye. It would be impossible to prove the existence of such a condition except in the event of the occurrence of the anomaly in one eye of an individual whose other eye was normal. Great misunderstanding of the theory has been evident in connection with such points.

72. **Special Cases of the Failing Colour.**—Young's supposition that the failing colour is a fundamental colour may correspond to the vanishing of the three co-efficients of one denomination, such as l_1, m_1, n_1. In this special case green and blue stimuli are alone effective. A solution of the problem of this kind could arise through paralysis of the red centre; and it might occur without the presence of any other abnormality, so that the equations could be

$$x'=x=0, \ y'=y, \ z'=z.$$

But Young's special type could also occur without paralysis of a centre or annulment of its stimulus. Thus if m_1, n_1, m_2, and n_2 all vanish, the requirement for no stimulation of the red and green centres is $x_0=0$. So the failing colour must lie upon the line forming the green-blue side of the triangle. And if, further, we have say $l_3=m_3=0$, the requirement of no stimulation of the blue centre is $z_0=0$. Then the failing colour coincides with the green corner of the normal colour triangle. In this simple case there may be no change from normality except in so far as the green centre is stimulated from the red retinal source of excitation and from that alone.

Again, if $l_1=m_1=\frac{1}{2}$, $n_1=0$, $l_2=m_2=\frac{1}{2}$, $n_2=0$, the red and green retinal stimuli each excite equally the red and the green centres. So, if the blue action is normal, the dichromatic eye perceives yellow and blue colours only. This is an example of the possibilities pointed out by Helmholtz after it had become apparent that cases of dichromasy existed in which simple lapse of one fundamental did not occur.

In the previous case the sheaf of lines of constant colour radiate from the green corner of the colour triangle towards the opposite side. In the present case these lines are all parallel to the red-green side of the colour triangle. The radiant point of the sheaf, i.e., the failing colour, lies at infinity in the direction of that side. For the condition $z'=0=z_0$ makes the failing point lie on the line forming the red-green side of the colour triangle; and the condition $x_0+y_0=0$ makes it lie also on the bisector of the external angle at the blue corner of the triangle.

73. **Differential Sensitivity to Colour.**—From the preceding considerations it appears that it is quite possible, using a set of normal but non-absolute fundamentals, i.e., a set each of which is a linear function of the absolute fundamentals, to find, out of the infinity of other such arbitrary sets which might equally well be used in accordance with the Newtonian law of colour mixture, one which will include a failing colour of dichromasy: so this possibility affords no evidence that the particular set having this property is the absolute set. To obtain any information of this point appeal must be made to a non-linear relation regarding colour vision should such exist. As Helmholtz pointed out, the law of intensity of sensation as dependent on intensity of stimulus, i.e., the law $dS=dx/x$ (§ 28) is a non-linear relation of the requisite kind: and his explicit recognition of the dependence of differential sensitivity upon this non-linear relation has been considered already in § 47.

To measure the differential sensitivity one of two identical spectrum colours may be gradually varied until a difference is just noticeable. Otherwise, a series of spectrum colours which have various small given differences may be grouped so as to include in each group colours which do not appear to be different, or which just appear to be different. In each method the statistical average, obtained from very many observations, must be taken. The results may then be exhibited diagrammatically by plotting the just discriminable difference of wavelength against the wavelengths themselves throughout the spectrum. Minimum and maximum differences then correspond to maximum and minimum sensitivity respectively.

But, since the difference between two spectrum colours

depends upon three independent qualities, intensity or brightness, hue or colour, and purity or saturation, we may have various types of curve in accordance with the nature of the test employed in any of the above methods. First, the difference of brightness alone might be considered. The curve would then be one exhibiting the variation in sensitiveness to luminosity. Second, change of colour alone might be taken into account: and this might be done under spectrum conditions of luminosity, or with the luminosity adjusted to equality at each wavelength. Third, by the addition of white

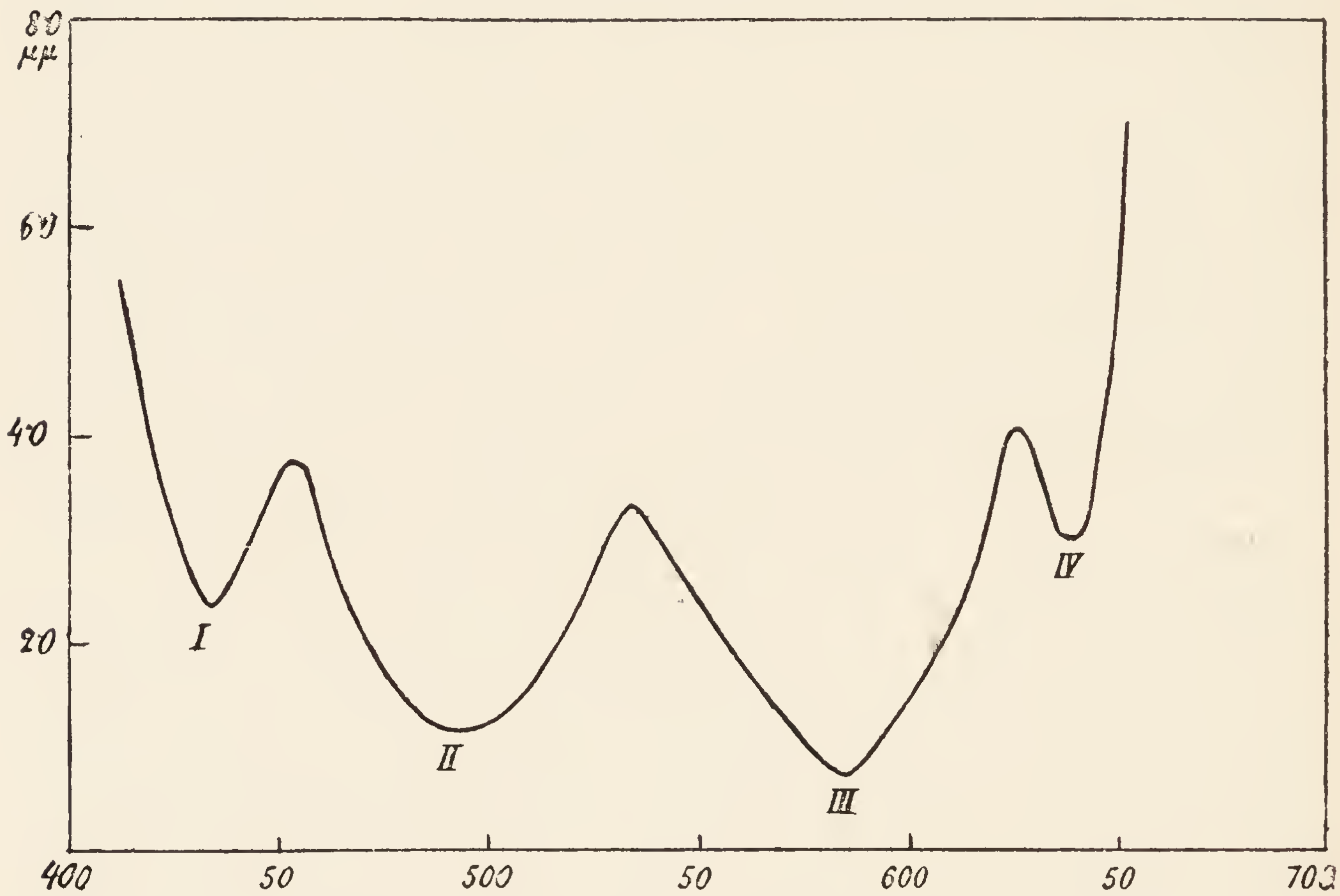

Fig. 18.—Curve of Differential Sensitivity.

light to the less white of the two near lights, an estimate of the difference in purity might be attempted. Again, the test might be general without alteration of any condition. Or, to take a final example, the difference of wavelength might be made, for each wavelength, absolutely the smallest observable, by change of the intensity of the light so as to secure the condition.

As might be expected from previous considerations, results vary to some extent even amongst ordinary trichromatic eyes. Several maxima and minima of susceptibility are found in the range of the spectrum. A curve given by Steindler for sensi-

tivity to change of hue is shown in Fig. 18. The one given by Helmholtz originally, from results of König and Brodhun, appears in Fig. 20. In that case the estimate of general sensitivity was made.

74. **Steindler's Test of Colour Sensitivity.**—In Fig. 19 a portion of the three equal-area curves is supposed to be represented over part of the visible range of wavelength. The stimulus c' is taken as the smallest. The white stimulus is therefore $3\ c'$; and $a'c'$, $b'c'$, respectively, are the two outstanding components of colour stimulus. If the ratio of the two varies with great rapidity as the wavelength alters, it is evident that the resultant colour impression will change rapidly as the wavelength alters; and conversely. The sensitivity

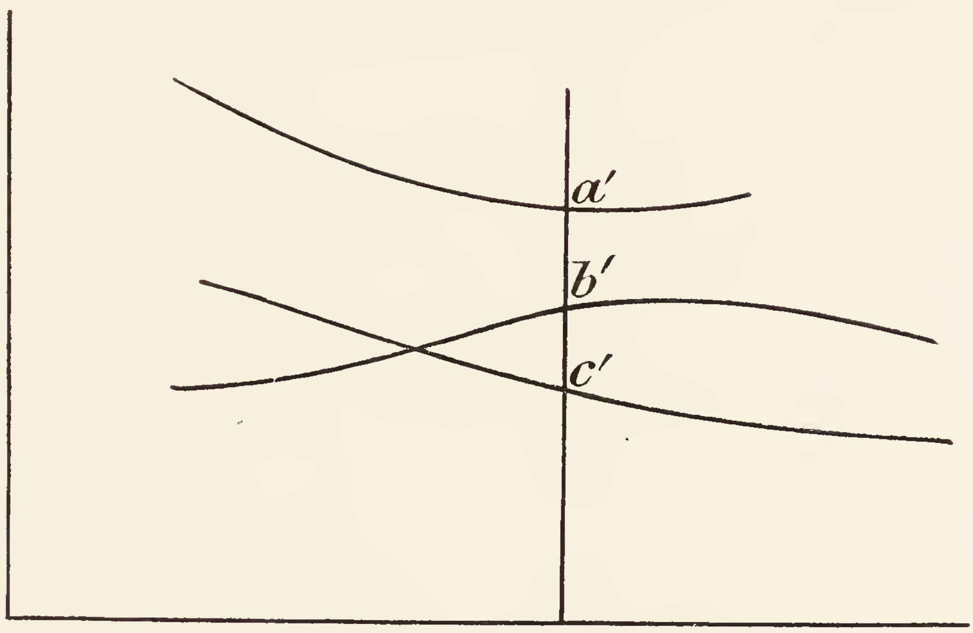

FIG. 19.—Test for Colour Sensitivity.

to hue variation will therefore be great or small accordingly. On this quite generally correct line of argument Steindler, using experimental data given by Exner, showed that the calculated positions of maxima and minima agreed very closely with those actually observed: and König pointed out the great value of the coincidence as a confirmation of the trichromatic theory.

It is not, in strict accuracy, the ratio of $a'c'$ to $b'c'$ that should be employed in the calculation. For, in accordance with the simplest form of Fechner's law, the intensity of the sensation remaining over when part of the stimulus of one fundamental is removed is given by

$$\log a'' = \log a' - \log c'.$$

Similarly

$$\log b'' = \log b' - \log c'.$$

Thus the remaining subjective stimuli, measured on the equal-area scales, are a'/c' and b'/c' respectively ; and the difference of the colour impressions is $\log(a''/b'') = \log(a'/b')$. So that the colour sensation changes with wavelength at a rate which is proportional to the rate of change of the fraction a'/b', and is also inversely proportional to the value of the fraction itself, provided that the fundamental sensations are mutually independent and additive, as they are individually.

75. **The Meaning of Threshold Values.**—The change in the sensation value a' above is

$$a' - a'' = a'\frac{c' - 1}{c'},$$

and vanishes whenever c', that is c/c_0, where c_0 is the threshold value and c is the stimulus, falls to the value unity.

Now, in this illustration, c' may be any sensation value of the type a provided only that it does not exceed a' in value. This very directly suggests that the threshold value is the value of the external stimulus which forms the least perceptible difference from the so-called self light. Behind this question lies the further one whether the threshold value is entirely due to the self light, so that it would vanish if the latter vanished ; or is in part due to structural or functional conditions, so that it possesses a small finite limit under extreme dark adaptation.

The identification of the threshold value of a compound light with the ratio of the sum of the component external stimuli to the sum of the sensation values permits the passage to the law of colour mixture, and finds its justification in connection with the law of white equivalents and the experimental determination of the curve of threshold values.

76. **Helmholtz's Treatment of Differential Sensitivity.**—Of the several modes of observation of sensitivity to differences in spectrum colours, the one discussed by Helmholtz was the one mentioned last in § 73. Before any strict discussion can be given it is necessary that an expression shall be found, or postulated, for the resultant sensation, or for its law of variation, in terms of the component sensations and their

variations with wavelength. In the development of knowledge on the subject, the procedure adopted may either be that of obtaining a wide experimental basis for the subsequent development of theory, or it may be that of postulating a simple law based on guidance given by present data and then proceeding to test the postulate by comparing its consequences with other known or subsequently acquired data. As he always did in his development of the subject of colour vision, Helmholtz followed the latter course. Each fundamental sensation possesses magnitude and a distinctive characteristic. It is therefore analogous to any directed quantity which possesses magnitude and a distinctive direction.

Helmholtz presumed tentatively that the analogy could be regarded as complete. Three mutually independent directed physical quantities of any kind can be represented by three mutually perpendicular lines, and their resultant is represented by the diagonal of the framework of which these lines form coterminous edges. Thus a change in the resultant magnitude is the square root of the sum of the squares of the changes in the component magnitudes. He took this law as giving the change of the resultant sensation when the changes of the component sensations are given in accordance with Fechner's law. This brilliant extension of Fechner's law regarding the intensity of light of one colour, so as to include in its range the variation of colour also, is one of the most beautiful examples of inductive inference to be found in the whole extent of physical science : and it achieved a wonderful success in his direct application of it to the theoretical determination of the law of differential sensitivity and the problem of the absolute fundamentals.

In continuation of the discussion in § 48, we have now his own further treatment of the subject.

"Regarded in connection with the above-mentioned problem, to compare the perceptivity for difference of brightness with that for colour difference, the hypothetical extension of the psychological law formulated by me, would, if found thoroughgoing, be able to solve a problem for the theory of colour perception, for which hitherto no very certain support has been given, namely the determination of the three active physiological simple colour sensations.

" We have seen that Newton's law of colour mixture certainly permits us to refer the complete manifoldness of colour perception to three juxtaposed modes of excitation of the optic nerve apparatus, but leaves quite or almost quite unsettled to which colour perceptions these three elementary excitations correspond. If we regard with Newton all the spectrum colours and their mixtures as represented in a colour diagram, the positions of the three ground colours in Young's theory would be subject to a single limitation only, that the triangle formed by them must include in itself the totality of the spectrum colours. If, on the other hand, we admit with E. Hering negative excitations, there would subsist no limitation on the choice of the three primary colours.

" This problem seems to me weighty enough for its solution to be attempted, in so far as that is possible with such observations, in many ways inadequate, as are up till now at our service, even if one dare only hope to obtain a provisionally approximate solution. At the same time it is necessary to show whether also the observations on the colour sensitivity of trichromatic eyes comply so far with our psychological hypothesis as is to be expected from the subsisting error limits of the observations.

" In the latter connection I point out here specially the present still subsisting insufficiency of the observations. According to rule great accuracy is particularly unattainable in all measurements at the limit where a phenomenon is yet perceptible ere it vanishes. The point in question here is the perception of the colour differences of neighbouring spectrum colours. Therein, as in nearly all similar cases, come into play all kinds of uncontrollable changes in the condition of our nerve apparatus and psychical activity, which become apparent finally in the variation of the results of measurement.

" In point of fact the comparison of colour tones in the latest measurements of A. König and E. Brodhun were made on equally bright seeming colours, and we can well assume that the two observers have sought to produce the most suitable brightness for this object. These fall in the region of validity of the normal Fechner's law, where the apparent steps in brightness are in proportion to the absolute light strengths,

But even if this can hold for the majority of spectrum colours, it is questionable whether departures from this simplest form of Fechner's law cannot enter in where one or two of the elementary colour impressions in the compound colour were very weak, e.g., in very saturated colours whose weak intermixture of other colours determines the colour difference. Such departures from the law can appear here as enter in in the case of small luminosity. In fact we shall meet departures of this kind between calculation and observation. If data regarding the absolute light strengths of the coloured fields under comparison were given, one would be able to calculate the greater insensitivity in the condition named, with regard to the colour difference in question. Indeed these variations in the ratios of the colour triangle cannot, as we shall find, be very large, since all spectrum colours exhibit themselves as having strong components of the ground colours.

"The numerical data which form the actual basis for the calculation under consideration have been obtained in various independent investigations which were carried through without any reference to the present object. Had the latter been the case some facilitation of the calculation and substantial security for its accuracy would have been attainable. In particular, the calculation is rendered difficult and the accuracy of the result notably prejudiced in that, on the one hand, the estimation of the mixture ratios of the colours, and, on the other hand, the estimation of the acuteness of vision for colour difference are not made usually on the same wavelengths, so that the numbers for the mixture ratios which are brought into the reckoning have to be found in part through interpolation. In fact, the simultaneously used values of the differential co-efficients of the colour values in the spectrum with regard to the wavelengths are generally only got through interpolation, and just at some points where the differential co-efficients change very rapidly, smaller intervals for the observations had become highly desirable.

"Since the numbers found by König, who, by the help of an empirical formula, had obtained by artifice calculations concerning the prismatic soectrum of gaslight from the interference spectrum of sunlight, exhibited undoubted small

irregularities in the intensity curve of the elementary colours constructed by him, it seems best to employ a graphical interpolation, as, moreover, the author mentioned has himself done in the curves published by him and C. Dieterici. This interpolation has been made by Dr. Sell, who has carried out the greater part of the highly wearisome calculations, and that at a time when neither he nor I could tell what influence the form of the curve would have on the form of the hoped for results of the calculation.

"There were sufficient observations for eighteen wavelengths. If one can assume that the simple first form of Fechner's law can be regarded as applicable throughout, six parameters have to be sought which will give nearly the observed values of the perceptive power of the eye. The equations from which the parameters have to be found were of the sixth degree in each, and so were only soluble by gradual approximations. Nevertheless there exist rules regarding the sense of the changes of perceptivity for single wavelengths occurring with changes of the single parameter which can serve as guides in the calculation.

"Finally, the calculation can in general only be so far carried out that the outstanding differences between calculation and observation exhibit no regularity, or at least none that was not already explained by the departures from Fechner's law. The great labour which it would have involved to diminish the difference farther by employing the method of least squares seems to me, in contrast with the insufficient accuracy of the basic observations, which can easily be improved in the future, to be unjustifiable.

"Since this investigation ought to exhibit a relation between quantities, for which hitherto such was absolutely unknown, and which, if the dependence presupposed by us between them, or an analogous one, does not exist, might have stood even so far apart as in the ratio of 1 : 100 or 1 : 1,000 instead of being approximately equal, it would always have to be regarded as a provisional result if the same, in spite of all mentioned unfavourable conditions, only departed from their mean value in the ratio of 1 to 1·5."

77. **The Absolute Fundamentals.**—Helmholtz then used the equation given last in § 47, written in the form

$$\frac{dS}{k}=\frac{d\lambda}{\sqrt{3}}\sqrt{\left(\frac{1}{x}\frac{dx}{d\lambda}-\frac{1}{y}\frac{dy}{d\lambda}\right)^2+\left(\frac{1}{y}\frac{dy}{d\lambda}-\frac{1}{z}\frac{dz}{d\lambda}\right)^2+\left(\frac{1}{z}\frac{dz}{d\lambda}-\frac{1}{x}\frac{dx}{d\lambda}\right)^2},$$

for the purpose of determining the absolute fundamentals. According to the theory the equation can only give correct values for the changes of wavelength $d\lambda$ corresponding to a given change dS in sensation, which is brought to a minimum by suitable regulation of the light strength, provided that x, y, z represent the absolute fundamentals. These unknown fundamentals are, in accordance with Newton's law, linearly related to any experimentally used set. Helmholtz employed the set used by König and Brodhun, and the corresponding

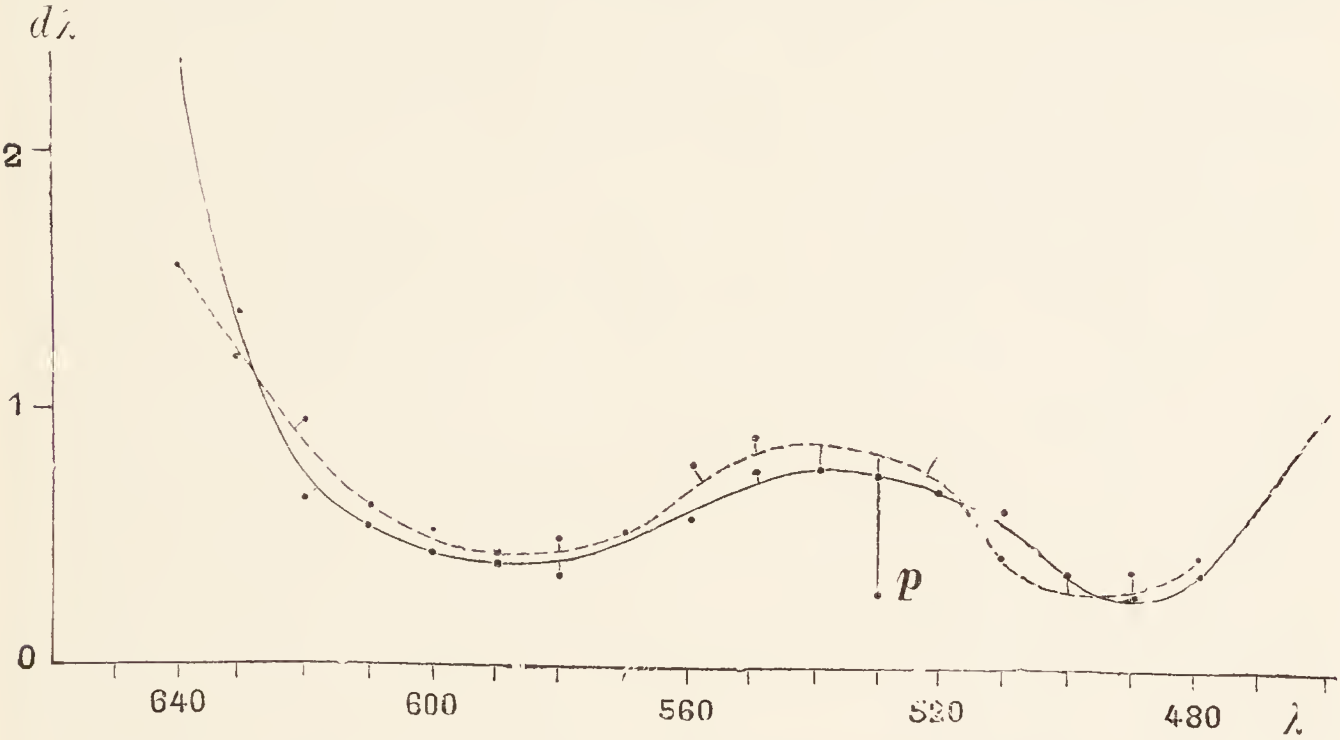

Fig. 20.—Helmholtz's Differential Sensitivity Curve.

numerical values. The nine quantities specifying these relations reduce to six since only the ratios of dx to x, etc., appear in the equation above. These are the six parameters referred to in the preceding section.

Fig. 20 shows the experimental curve in the full line, and the theoretical curve in the dotted line. The ordinates are values of $d\lambda$ which correspond to a just perceptible difference of sensation, and the abscissæ give values of λ. Under the conditions of uncertainty described in § 76, the success of the prediction of the variation of differential sensitivity in obedience to the laws of trichromasy is remarkable. Its brilliance is of the same order as that of the prediction by Newton of the

value of the moon's acceleration in obedience to the law of gravitation ; or of the prediction by Maxwell of the speed of propagation of light in obedience to the laws of electromagnetic action.

No slight modification of postulates, should these be necessary, can lessen the greatness of this pioneer work.

CHAPTER X

SENSATION VALUES AND THE SPECTRUM CURVE

78. **The Directly Perceived Aspects of Sensation.**—The power which the perceptive apparatus possesses of discriminating as to the whiteness of coloured lights is, in large part at any rate, the basis of the postulate, sometimes made, that whiteness is a fundamental perception. The inessentiality of this view, in so far as mere formal statement is concerned, appears at once from the consideration that, in an arbitrary linear transformation of fundamentals, one of the new fundamentals can be chosen as white. Indeed the trichromatic theory at once suggests that a deviation from the condition of equal subjective stimulation of the three fundamentals would be appreciated as a deviation from normality. And it shows further that this deviation, in all its apparent complexity, involves only two other variables at any one wavelength throughout the whole range of visual perception. In a case of dichromasy the necessity for the restriction to any one wavelength disappears. A case of anomalous trichromasy, described in § 44, occurs if the derived white fundamental has actual existence. In the general case the two variables additional to white may be the colour value and the luminosity value, each of which is some function of the three fundamental stimuli.

The three attributes whiteness, colour, and luminosity constitute the directly perceived aspects of visual sensation. Their resultant effect is that of the total sensation. Helmholtz's great generalization enables us to represent the latter, S, as the square root of the sum of the squares of the three fundamental sensations, so that

$$S^2 = S_r^2 + S_g^2 + S_b^2,$$

or, in differential notation, as used by him in his prediction of the law of differential sensitivity throughout the spectrum,

$$(dS)^2=(dS_r)^2+(dS_g)^2+(dS_b)^2.$$

The first equation can be expressed in the form

$$\log^2\frac{c}{c_0}=\log^2\frac{r}{r_0}+\log^2\frac{g}{g_0}+\log^2\frac{b}{b_0},$$

and the second in the form

$$\left(\frac{dc}{c}\right)^2=\left(\frac{dr}{r}\right)^2+\left(\frac{dg}{g}\right)^2+\left(\frac{db}{b}\right)^2$$

which does not involve the threshold values, and so is directly usable.

79. **Dependence of the Direct Aspects on the Fundamental Stimuli.**—The case of the dependence of the sensation of luminosity upon the stimuli has been discussed in § 67. It was there found that some support existed in favour of the supposition that the effective external stimulus in that sensation is the arithmetical sum of its fundamental components. The postulate that the sensation value of the total stimulus of luminosity is the arithmetical sum of the component sensations values, was found to give an expression for the threshold value of the luminosity which is in general accordance with observation ; and it also led to Abney's experimentally determined law of summation of luminosities. That experimental support formed the only justification for the use of the symbolic equation

$$\frac{c}{c_0}=\frac{r}{r_0}+\frac{g}{g_0}+\frac{b}{b_0}$$

as an algebraic equation. The algebraic relation amongst the quantities can only be written in the form

$$c'=\frac{c}{c_0}=\mathrm{F}\left(\frac{r}{r_0},\ \frac{g}{g_0},\ \frac{b}{b_0}\right)=\mathrm{F}(r',\ g',\ b'),$$

where the function F is determinable by experiment alone, as also is the function c_0. When we deal with colour sensation, the threshold values must be those of colour perception.

In the case of the resultant sensation, Helmholtz's reasoning leads to the relation

$$\log^2 c''=\log^2 r'+\log^2 g'+\log^2 b' \quad . \quad . \quad . \quad . \quad \text{(A)}$$

and he attempted its experimental justification in the differential form with distinct success.

The question now arises, What is to be taken as the correct representation of the sensation of colour? On the trichromatic view colour depends essentially and exclusively upon the ratios of r', g', and b'. And it vanishes when these ratios are unity. These conditions suggest at once that, in the estimation of colour, we should assume (§ 60) the law

$$\log c''' = \log r' + \log g' + \log b' \quad . \quad . \quad . \quad . \quad (B)$$

in so far as the ratios involved in colour sensation, pure or impure, are concerned.

We may conveniently use the generalized expression

$$\log c''' = \log \xi + \log \eta + \log \zeta,$$

where $\xi \nless \eta \nless \zeta$. With ζ itself, an equal amount of each of the other two internal stimuli ξ and η contribute to produce the sensation of whiteness. So the resultant white sensation is, by the above law,

$$\log w = 3 \log \zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad (C)$$

And the pure colour sensation is

$$\begin{aligned} \log \gamma &= \log \xi + \log \eta + \log \zeta - 3 \log \zeta \\ &= \log \frac{\xi \eta}{\zeta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (D) \end{aligned}$$

80. **The Field of Experimental Inquiry.**—The only direct test of any point in the above scheme is that supplied by Helmholtz himself in his discussion of the determination of differential sensitivity throughout the spectrum. A few other more recent experimental measurements are available; but, in the main, this experimental field is unexplored. An immensely wide experimental basis is required for the settlement of the problems of colour vision involved. Exhaustive tests of many normal and approximately normal eyes, under conditions made as standard as possible, have to be made. In particular, standard spectrum fundamentals of definite wavelengths should be chosen; and both sets of the sensation curves should be found in terms of these. Intensity should be made standard for some definite wavelength. Peculiarities of vision, e.g., extent of spectrum, and boundaries of main colour stretches, should be noted, and luminosity curves found.

Threshold values throughout the spectrum should be determined under different definite conditions ; and in particular, differential sensitivity tests are of the highest importance. These should be made under various conditions.

The conditions might be those (1) of the normal spectrum, so as to obtain the total sensitivity ; (2) of the normal spectrum with adjustment of intensity, as in the work of König and Brodhun used by Helmholtz, so as to secure the smallest detectable step ; (3) of addition of white light to the more saturated of the two lights under comparison ; (4) of adjustment to a common intensity ; (5) of adjustment to a common whiteness or saturation ; (6) of adjustment to a common intensity and whiteness, that is to say, with variation of colour alone. Again, comparison may be made between two lights of any one wavelength ; (7) with variation of intensity alone ; or (8) with the saturation alone altered.

81. **Relation between a Sensation Curve and the corresponding Curve of Differential Sensitivity.**—When a sensation curve, whether it be that of the resultant sensation, or of colour, and so on, is known, the differential curve can be found by plotting the slope $dS/d\lambda$ of the sensation curve at any wavelength against that wavelength. But, in the direct determination of the differential sensitivity, the quantity that is found is the magnitude of the change of wavelength which gives rise to a just recognizable sensation, which is regarded as a constant quantity.

If the sign of the change were also considered, the curve obtained by plotting the change against the wavelength would be the reciprocal of the former curve, but there is a deviation from this in consequence of the change being measured regardless of sign. The parts which lie normally on the negative side of the line of wavelengths are plotted instead on the positive side.

Every maximum on the sensation curve implies zero change of sensation for a small increase of wavelength. Therefore at these points the change of wavelength per unit change of sensation is infinite—positive on one side of the stationary point, negative on the other. As a result of the negative values being plotted as positive there is a maximum on the differential curve wherever a maximum or a minimum occurs

on the sensation curve. The minima on the differential curve occur at wavelengths which give points of contrary flexure on the sensation curve. Thus the four minima on Steindler's curve require four contrary flexures on the sensation curve. And the three maxima on Steindler's curve necessitate two maxima and one minimum on the sensation curve since its ordinates vanish at the ends of the spectrum.

The maxima on the differential curve are theoretically infinite, but in practice they are finite because of the impurity of the spectrum. A curve similar to Steindler's is shown in

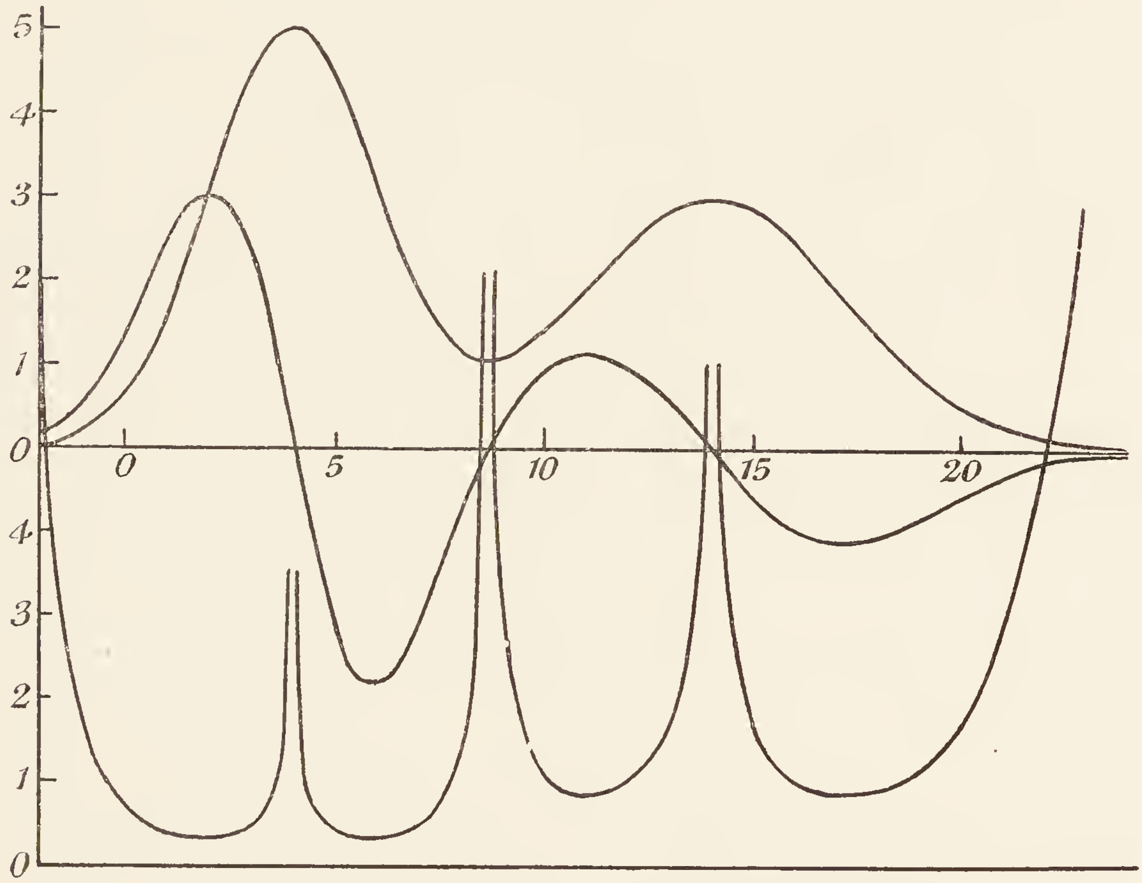

Fig. 21.—Relation of the Curves of Sensation and Differential Insensitivity.

the lower part of Fig. 21, and the corresponding sensation curve is shown in the upper part. To give two side minima, as in Steindler's case, the sensation curve must be concave to the line of wavelengths at its two ends. To give two intermediate differential minima, the sensation curve must have two maxima. If the sensation curve were that of luminosity, rising from zero at either end through points of contrary flexure to a central maximum, the curve of differential sensitivity must have a central maximum with two side minima beyond which it rises continuously.

It is to an analysis of curves of differential sensitivity that we must look for a discrimination amongst the different types of curves of sensation.

Fig. 21 is illustrative only. The sensation curve has the equation

$$X = 5\varepsilon^{-\frac{1}{2}(\lambda-2)^2} + 3\varepsilon^{-\frac{1}{5}(\lambda-7)^2}.$$

The oscillating curve is that of $dX/d\lambda$ and is the curve of differential sensitivity, if the negative ordinates are plotted as positive. The lower curve is found by plotting the reciprocals of the ordinates of the differential sensitivity curve. The three central crests extend to infinity, but, in plotting from experiment, they are truncated as we have just seen.

The general resemblance of this sensation curve to that of Helmholtz's curves (p. 66) of approximate absolute colour sensation, to the curve of total sensation value xyz, and to the curve of white sensation value ζ^3, where ζ is the smallest of x, y, z, is very strong. Steindler's curve confirms Helmholtz's.

82. **Test of the Postulated Law of Pure Colour Sensation.**—We may regard the result of Helmholtz's test of equation (A), § 79 (in a modified differential form), exhibited in Fig. 20, p. 130, as a successful vindication of the approximate accuracy of the expression. A higher approximation could be obtained by using the modified form of Fechner's law. The modification of the equation which was used was the one necessitated by the experimental conditions under which König and Brodhun worked in their determination of the differential sensitivity. These were different from the conditions employed by Steindler. But Steindler's curve agrees in its central region very well with Helmholtz's curve. She used the ordinary spectrum conditions, in which intensity and purity as well as colour varied from point to point, and measured the change of wavelength giving a just perceptible change of colour. It is not easy to say if or how the estimate may be affected by the presence of the other qualities.

The upper curve in Fig. 22 represents the value of

$$\log\frac{\xi\eta}{\zeta^2}$$

(eq. D), where ζ is, at any wavelength, the numerically smallest

of the three fundamentals chosen by Helmholtz so as to get good correspondence between the observed and the theoretical curves in Fig. 20, ξ and η being the other two fundamental stimuli. It is to be observed also that ξ, η, and ζ are given in such units that equal amounts of the three are present in white light. The subjacent curve represents by its ordinates one-tenth of the quantity $\xi\eta/\zeta^2$.

It is a fact of importance that the general trend of the two curves, the true sensation curve and the curve of the sensation values, is so similar.

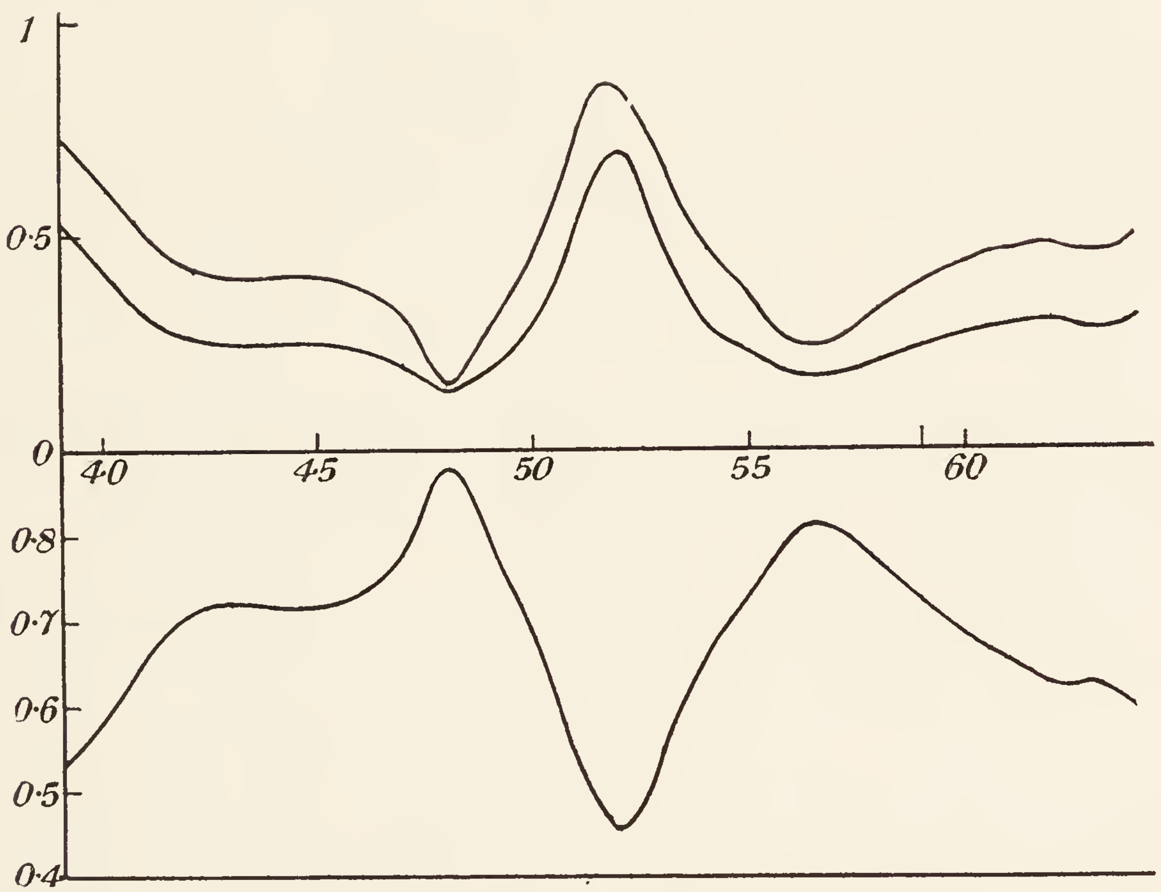

Fig. 22.—Curves of Colour Sensation, Sensation Value, and Fractional Whiteness.

The correspondence to Steindler's and Helmholtz's curves is very noticeable, stationary slope corresponding to stationary ordinate. There is a central maximum with two strong minima, and two side maxima and minima are also apparent.

83. **The Colour Sensation Value and the Fractional Colour Stimulus.**—The lower curve in Fig. 22 gives the value of the quantity

$$\frac{3\zeta}{\xi+\eta+\zeta}.$$

This, on the trichromatic theory, represents the fraction of the stimulus which corresponds to the impression of whiteness in the observed light. The inverse correspondence of the form of the curve to that of the intermediate curve is very clear. Consequently the curve obtained by plotting the quantity

$$1-\frac{3\zeta}{\xi+\eta+\zeta}$$

against wavelength must exhibit a strong similarity to the curve showing the value of $\xi\eta/\zeta^2$. The closeness of the relation

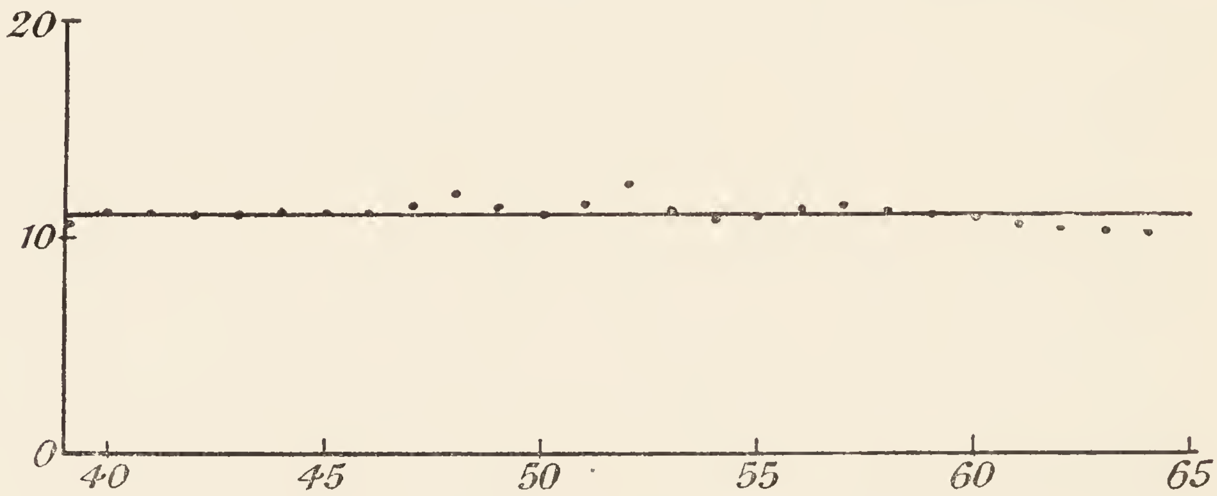

FIG. 23.—Relation giving the Spectrum Equation.

is made evident in Fig. 23. The ordinates give the values of

$$\frac{\xi\eta}{\zeta^2}+12\frac{3\zeta}{\xi+\eta+\zeta}$$

at wavelengths which are indicated along the line of abscissæ.

The maximum deviation from the mean value, 11·14, amounts only to 1·2 per cent., while the variations of the component terms from their respective means amount to 77 per cent. and 30 per cent. respectively.

We may therefore write the relation in the form

$$\frac{\xi\eta}{\zeta^2}=12\left(1-\frac{3\zeta}{\xi+\eta+\zeta}\right)-0{\cdot}86.$$

The result suggests a possibility, not real however (§ 84), that the small constant term should be zero. If that were strictly so, the fraction of the subjective stimulus which corresponds to the impression of colour is strictly proportional to the colour sensation value $\xi\eta/\zeta^2$. But it is that fraction

which is normally taken, on the trichromatic theory, as representative of the colour value of light.

This approximate correspondence gives experimental support to the assumption that the true colour sensation value is given by (B). It is perhaps not impossible that the slight observed deviation indicates a want of entire correspondence between Helmholtz's fundamentals and the true absolute fundamentals which alone, as he observed, can satisfy equation (A), § 79.

84. **The Spectrum Curve.**—We can use the experimentally determined relation

$$\frac{\xi\eta}{\zeta^2}+36\frac{\zeta}{\xi+\eta+\zeta}=11{\cdot}14$$

to find the form of the spectrum curve in the colour diagram. From the equation of the colour plane, $\xi+\eta+\zeta=1$, this becomes

$$\xi\eta=11{\cdot}14\zeta^2-36\zeta^3,$$

and, from the two equations, we can find the positive and real values of ξ and η which correspond to any value of ζ, not greater than $1/3$. The result is shown in the two upper parts of the colour diagram in Fig. 24. The spectrum curve consists of two branches, an inner and an outer, each exhibiting equilateral symmetry. The three corners of the triangle are isolated points on the curve. Two sides only of the inner curve are represented.

It must be supposed that, on the presumption of the suitability of the formula D, § 79, the inner curve is that of the subjectively active stimuli. As we have seen, § 38, the subjectively effective region in the colour diagram includes, and extends more or less, under conditions of fatigue or contrast (pp. 148–154), outwards from it. The outer curve must be supposed to lie in general outside it. Whether or not a part of it, in the neighbourhood of its least distance from the inner curve, may lie within the limits of extension of the visual field in those cases of dichromasy in which the yellow or orange light excites a specially powerful perception, is a matter of conjecture.

The strong similarity of the inner (full) curve (the lower side being ineffective) to the spectrum curve, especially that

of Helmholtz, is very obvious. The formula (B) compels equilateral symmetry in the spectrum curve ; but it must be recognized that it is liable to deformation by physiological peculiarities, such as absorption. If the formula is correct, we must regard the absolute fundamentals, to which it must refer in consequence of the non-linearity of the expressions used in obtaining it, as differing somewhat from Helmholtz's approximate determination.

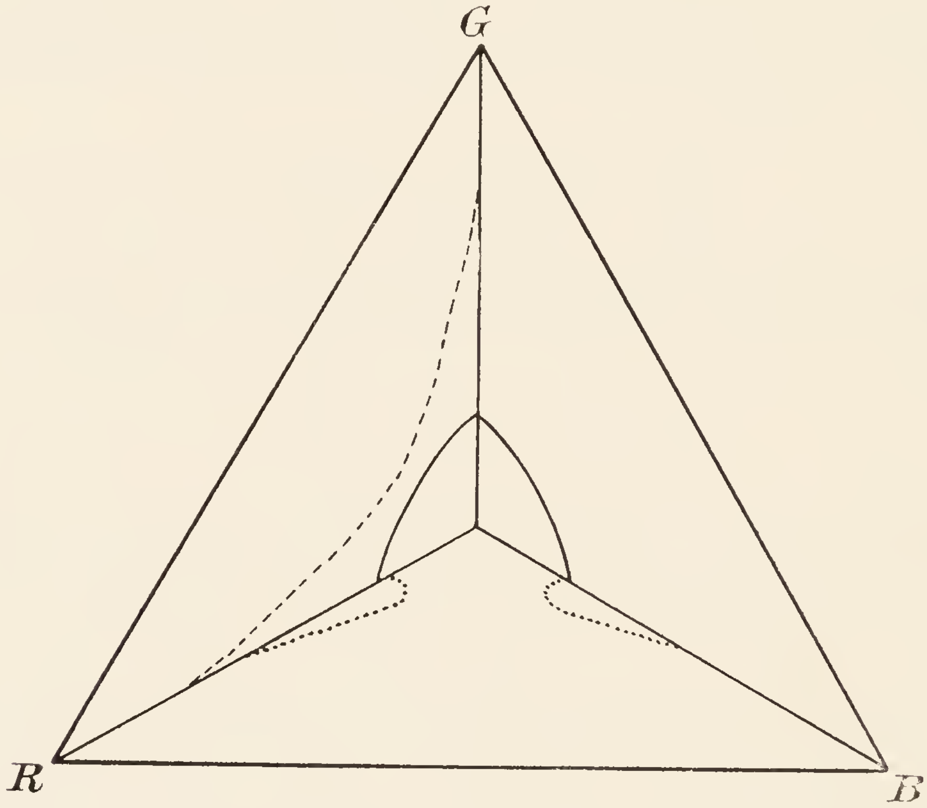

Fig. 24.—The Spectrum Curve in the Colour Diagram.

Much more extensive observations are required before these questions can be fully solved.

The lower (dotted) part of the diagram in Fig. 24 shows the nature of the spectrum curve if the small constant term in the last equation of § 82 were omitted. The two branches have coalesced ; and then broken apart transversely so as to exhibit three loops, of cardioid shape, related to the three corners of the triangle. It is entirely unsuitable. So, if the law D is correct, we must regard the normal use of the colour fraction to represent the colour value as only approximately correct.

CHAPTER XI

LOCAL AND CONTINUED ILLUMINATION

85. **Space Variation in Retinal Conditions.**—The optically sensitive part of the retina is located in the "rod and cone" layer. In the very small central region of clearest vision, and best appreciation of form and colour, the fovea centralis, cones alone are found, though these are more rod-like than the cones of other regions. The yellow spot, or macula lutea, extends throughout a central region several times larger in linear dimensions than the fovea. The rod-free condition of the fovea persists through the more central part of the yellow spot; but gradually cones are replaced by rods until, in the extreme peripheral parts of the retina beyond the yellow spot few cones are found. Appreciation of feeble illumination is strongest in the peripheral parts.

The chemical substance rhodopsin, or "visual purple," found in association with the rods at least, is decomposed by the action of light. It is specially noteworthy that its curve of absorptive power for light throughout the spectrum is coincident in general form with the curve of luminous persistency, that is, of luminosity in the neighbourhood of perception of light (§ 66).

In close correspondence with the regional distribution of structural conditions in the retina lies the regional variation of visual phenomena.

The extreme peripheral parts of the retina are nearly or entirely blind to all colour. The central part of the retina, where cones lie exclusively or in preponderance, possesses in normal eyes full trichromatic power. It is to be expected therefore that the field of vision for white light exceeds that for coloured light; and this is the case. The region immediately within the monochromatic belt exhibits yellow-blue

dichromasy. Within that again lies a region in which red and green colours are perceived. The magnitudes of their fields differ somewhat, and the order of the two fields in respect of magnitude is variable. Of all the spectrum colours violet has the least extent of field.

The degree of saturation of the coloured light, its intensity and the dimensions of the retinal image, have considerable effect upon the extent of a field of colour. Indeed, when these are made large enough, colour is perceptible nearly if not quite at the periphery.

All these visual phenomena are influenced by the nature of precedent illumination.

86. **Time Variation in Retinal Conditions. Adaptation.** —When the retina has been subjected for some time to a given state of illumination, the sense impressions are in general, and sometimes very intensely, different from those in which the immediately precedent illumination has been of another type; and this is true of the impressions both of luminosity and of colour. Thus, after exposure of the eyes to outside light in strong sunshine for a little time, few objects can be seen on entry into even a well-lit interior, because of a luminous haze or glare which seems to fill the field of vision. The haze gradually fades away and the state of vision seems to be that of normal type; but exact tests show that the disturbed state can persist for a very long time. Conversely, on passing from a dimly-lit interior into strong outside sunlight, vision is greatly impaired or even painful and impossible for some time.

The process by which equilibrium between the stimulus and the sensation is attained is called the process of Adaptation—light adaptation or dark adaptation according as it has been carried out in light or in darkness more or less. We are in the habit of speaking, in colloquial language, of the eye as becoming "accustomed to the light" or "accustomed to the darkness." And we know it, especially in the latter case, as a condition which requires a considerable time for its full development. In dark adaptation it is usual for the condition to develop slowly for a few minutes, then rapidly for about half an hour, and finally very slowly. The characteristics of the three stages, slow, rapid, and again slow, suggest the

passage of some molecular action through conditions of group instability, as in the magnetization of iron.

Great individual variations exist in the rate of approximation to the final state, and the amount of the increase of the retinal sensitivity which is acquired. The effect appears in cases of dichromasy and monochromasy as well as in normal vision. In cases of "night blindness," in which the action of the retinal rods is non-existent or weakened, there may be little change in many hours, although normal sensitivity may be reached in about one hour in other cases.

In the central fovea the sensitiveness to weak light is usually very small. Thus it is well known that faint stars which can be seen by indirect vision disappear when the gaze is direct. The natural conclusion is that the cone structure, which is effective in direct vision, is not responsive to very feeble illumination. This might be due to inherent structural or functional conditions. On the other hand, it may be due to excitation, not necessarily visible, of self light in the foveal region; which, in accordance with Fechner's modification of his law of the sensation of luminosity, effectively lessens the sensitivity to illumination.

This latter view indicates the least complicated mechanism for the gain or loss of adaptation, whether that mechanism be located in the retina or in the brain. And it introduces no new aspect into the formal expression of the relations between stimulus and sensation, so that the laws of adaptation follow without additional postulates in the Young-Helmholtz theory.

A single observation upon a faint star seen against a dark sky, after hours of rest in entire darkness when the changing self light of the eye is distinctly visible, and the usual condition of faintness or invisibility by direct vision combined with good visibility by indirect vision, holds at first, but is then exactly reversed (*Proc. R.S.E.* 1905), is sufficient to indicate the probability of the reality of this explanation. It is impossible to describe the feeling of power in vision of a faint object which accompanies the addition of foveal clearness to the mere perception of luminosity.

It is quite in consistence with this view that the slight possibility of foveal dark adaptation which is normally observ-

able must be preceded by strong light adaptation. This accentuates the self light above its normal value.

The effect of dark adaptation upon colour perception is very marked. All sensation of colour ultimately disappears, and vision, even foveal vision, becomes essentially monochromatic, or rather achromatic like normal peripheral vision. The term Twilight Vision is frequently used to indicate the state of extreme dark adaptation in which the intensity of the illumination falls below the threshold of perception for any colour. Chromatic Twilight Vision occurs before the final stage is reached.

87. **Fatigue and Dazzle.**—When light falls upon the retina steadily the sensation evoked does not retain its maximum value. It gradually diminishes, and, under conditions of suitable intensity and duration of stimulus, may reach a very low value. This phenomenon is known as Fatigue. It is evidently an aspect of light adaptation; and its converse the Recovery from Fatigue, or Defatigue, is an aspect of dark adaptation. It is the time change of fatigue or of defatigue which constitutes the process of adaptation.

Since the sensation due to an external stimulus x is given by the expression

$$\log\frac{x}{x_0},$$

where x_0 is the threshold value of the stimulus, it appears that the departure of the threshold value from its minimal amount is a measure of the amount of fatigue. The phenomenon occurs with coloured as well as with uncoloured light; and thus the phenomena of adaptation and fatigue are, on the trichromatic theory, entirely determined whenever the three fundamental threshold values r_0, g_0, b_0 are known as functions of the time, the intensity of illumination, and other quantities. So far as the effects are concerned there is no need to settle the question of the origin of the threshold values—whether they correspond to the components of real self light, or retinal stimuli; or whether they originate in cerebral conditions (see end of § 94).

The investigation of colour fatigue shows that these three functions are sufficient for complete description of the pheno-

mena ; and thus the field of application of the theory is extended through a new and wide range.

Since self light is normally visible against a sufficiently darker background, it is to be expected that the self light induced by the process of fatigue will be consciously superadded to the light which falls upon the retina from objects regarded by the fatigued eye, provided that they are not too luminous. The effect is readily noticeable when the fatiguing light is coloured. The analogy to the known effects of mechanical fatigue is evident, and furnishes the reason for the employment of the term in dealing with the perception of light. A metal beam is at once visibly bent under the application of sufficient stress. But the amount of bending slowly increases towards a maximum under continued application of the same stress. Unbending occurs suddenly on the removal of the stress. The fatigued condition persists for some time, but gradually relaxes.

The relaxation from fatigue to violet light occupies a longer time than that induced by red light ; and, generally, fatigue by light of longer wavelength disappears sooner than fatigue by light of shorter wavelength. There is evidence that it also appears more quickly. After excessive fatigue by strong sunlight, all colour disappears from the landscape for a period, and then gradually the warmer tints reappear, the bluer tints coming back last. The fatigue colour may persist for weeks at the area of the retina upon which the direct image of the sun fell : indeed the test might easily be dangerous to vision.

When the fatigue tint becomes noticeable, the condition of dazzle arises, though the term is generally employed only when the induced self light becomes strongly noticeable. In this case, as in less marked cases, the threshold value of any light has become large. If the effective intensity x drops sufficiently below the temporary threshold x_0, the fatigue or dazzle tint may become manifest even against a background of the same colour, but it is more evident against a background of a different colour.

When the intensity of illumination of an object seen through self light or dazzle light is reduced so as not to exceed the intensity of the latter by more than the limit of differential perception, the object ceases to be visible.

88. **Fatigue by Spectrum Colours.**—In powerful fatigue of the eye by spectrum colours Burch found that fatigue with light having one colour predominant does not affect the strength of the remaining sensations. This is in agreement with the Young-Helmholtz postulate of the independence of the fundamental sensations. He showed also that each dazzle tint develops and dies down at a rate characteristic of itself ; red, green, blue, and violet tints disappear in the order named, i.e., in the order of diminishing wavelength. The luminous dazzle haze tinges all colours except that to which it belongs, provided that the brightness of these colours be not too great. The development of the dazzle haze illustrates the basic physical principle that if, with two directly related quantities, increase of the one gives rise to increase of the other, independent increase of the latter gives rise to decrease of the former.

Burch points out four spectrum colours which react in a special way towards strong fatigue. These are red, green, blue, and violet. Fatigue by red light blots out the red end of the spectrum, and the green extends in the direction of increasing wavelength up to the line C. Similarly, with careful fatigue by strong violet light, the violet end of the spectrum is shortened and blue replaces the remainder. Fatigue with strong green light causes the red and blue regions to extend towards each other and either meet or have a dark band of separation. Similarly, in fatigue with strong blue light, the green and violet colours extend towards each other and either meet or are separated by a dark shadow. Dazzling with orange light gives two dazzle tints, red and green, of which red disappears soonest and leaves green. Similar results held with other intermediate colours.

This constitutes a practical demonstration of the co-existence of fundamental positive sensations in accordance with Young's postulate.

Edridge-Green and Marshall have also investigated fatigue by spectrum light, and have found results differing in some respects from those got by Burch, especially in connection with yellow light. After fatigue with light of the sodium flame, spectrum yellow vanished and only a faint orange band separated the red and green regions. With prolonged fatigue, the red and green regions met. Red was unshortened in extent

and seemed to be more purple, while green became more blue; and the blue and violet regions became less bright. The magnitude of the fatigue was much greater in the case of Burch's work than in the other. Consequently no necessary contradiction between the two sets of results arises. The result that red and green seem to be more blue on fatigue with yellow light, and that there is no shortening of the red end of the spectrum, follow at once if both the red and the green sensations are fatigued by the yellow light, while all three sensations, red, green, and blue, are present to some extent at all visible wavelengths. But this is precisely the condition indicated in Helmholtz's analytical determination of the three absolute fundamentals and strongly supported by observations on differential sensitivity. Here then we have further evidence of the approximate accuracy of the determination, and of the validity of the trichromatic theory. But this evidence, furnished by the observations of Edridge-Green and Marshall, does not stand alone; their further observation of the diminution of intensity of the blue and violet after fatigue with yellow light is again just what is indicated as a necessary result by Helmholtz's determination of the absolute fundamentals. For not only is the red fundamental strongly stimulated in that region, but so also is the green. Therefore these components of sensation in the blue and violet regions are largely deleted by the fatigue, and so the intensity falls. These verifications of the trichromatic theory are fine examples of its predictive power in regions not explicitly contemplated when it was framed.

Porter and Edridge-Green have also experimented on fatigue by spectrum light, with results which are also in consonance with the trichromatic theory; but consideration of these requires discussion of after images.

Every additive result of theory, which is predicted by means of one triple set of fundamentals, follows necessarily by the same process when applied to any other set which is linearly related to that one. Thus all additive results, confirmatory of the trichromatic theory, which have been obtained by workers such as König, Abney, and others, must hold also when the process of deduction is applied to data given by a linearly related set of fundamentals framed to fit differential sensitivity

curves and Helmholtz's expression for the resultant sensation. In particular, Abney's results on fatigue for spectrum light, by means of spectrum light, are deducible, in his manner, from the approximately absolute curves determined by Helmholtz. It is the converse statement which is not true. Abney's fundamentals used in Helmholtz's procedure, which deals with non-linear expressions, would not lead to the observed trend of differential sensitivity throughout the spectrum.

In the deduction of fatigue effects by Abney's process, it is presumed that each component stimulus, i.e., ordinate of a sensation curve at a given wavelength, is diminished by fatigue in proportion to its magnitude. The curves are drawn on the equal area plan (§ 67), so that the postulate applies with the condition that white light remains white as fatigue proceeds.

Let f_λ be the proportionate reduction of stimulative power under given conditions with fatiguing light of wavelength λ, per unit stimulus. Then $f_\lambda x_\lambda$ is the proportionally reduced value, per unit stimulus, in the x-component when the light λ' falls on the eye fatigued by the light λ. Thus ${}_\lambda f_{\lambda'} x_{\lambda'} = f_\lambda x_\lambda x_{\lambda'}$ is the observed x-component when light of wavelength λ', having a normal x-component $x_{\lambda'}$, falls on the eye fatigued by the light λ. The ratios of the fatigued x, y, and z components are therefore $x_\lambda\ x_{\lambda'}\ :\ y_\lambda\ y_{\lambda'}\ :\ z_\lambda\ z_{\lambda'}$. The position of this light on the spectrum in respect of tone can therefore be found from the sensation curves. In so far as the presumptions are exactly valid, it follows that the order of occurrence of the two illuminations is without effect on the result. In example of the consequence of precedent fatigue by one light upon the apparent colour of another light we may use the sensation curves of Fig. 8, § 43, which represent the approximately absolute fundamentals deduced by applying Helmholtz's linear transformation to König's experimentally deduced fundamentals. At the wavelength $540\mu\mu$ the ordinates of the red, green, and blue curves are respectively 2·8, 6·5, and 3·8. At the wavelength $575\mu\mu$ the values are 4·25, 5·75, and 3·35 respectively. Hence the apparent colour of either light after fatigue by the other is that of the light in which the sensation ratios are about 12 : 36 : 13. That is to say, green is in excess, and red and blue are of about equal strength, the blue preponderating slightly. This condition corresponds to light of wavelength $560\mu\mu$

approximately. It exhibits the tinging of yellow illumination with green in consequence of previous illumination by green light. It may be visible or invisible in accordance with the intensities of the two lights which are involved in the functions ${}_{\lambda}f_{\lambda'}$ and f_{λ}.

89. **Theoretical Treatment.**—It is desirable to express the action in terms of the threshold values. The sensation values of the x-component of the first illumination before and after fatigue are given by

$$x'_1=\frac{x_1}{x_0},\quad x''_1=\frac{x_1}{{}_1x_0},$$

where ${}_1x_0$ is the threshold value as changed by fatigue. It is a function of the intensity, the wavelength, the duration and possibly the areal distribution of the illumination, even if it be made monochromatic. Thus

$$\frac{x''_1}{x'_1}=\frac{x_0}{{}_1x_0},$$

with similar expressions for the y and z components. The apparent colour of the fatiguing light will therefore be altered or not according as the three threshold values are altered in different proportions or in the same proportion. Absence of colour in white fatigue indicates uniform proportionality. But the three ratios, although always equal to each other with a given illumination, may alter together as the wavelength alters.

If now a fresh illumination, indicated by the suffix 2, falls upon the fatigued retina, the x sensation value normally produced by it,

$$x'_2=\frac{x_2}{x_0},$$

will be altered to

$${}_1x''_2=\frac{x_2}{{}_{21}x_0}.$$

So that the fractional reduction is

$$\frac{{}_1x''_2}{x'_2}=\frac{x_0}{{}_{21}x_0}.$$

Thus the second colour $x'_2+y'_2+z'_2$ becomes

$$\frac{x_0}{{}_{21}x_0}x'_2+\frac{y_0}{{}_{21}y_0}y'_2+\frac{z_0}{{}_{21}z_0}z'_2.$$

It is of interest to express the limitations imposed upon these co-efficients by the process of calculation adopted by Abney. His postulate gives three equations of the type ${}_1x''_2=px'_2\cdot x''_1$ with the same value of p in each. This gives

$${}_{21}x'_0=\frac{1}{px'_1}\,{}_1x_0,$$

with similar expressions in y and z. His postulate, therefore, implies that the threshold values of the sensation components, as determined by fatigue, are altered by the incidence of the second light in inverse proportion to the magnitudes of the fatigued components themselves.

The whole subject requires elaborate experimental investigation (see further, Chapter XIV).

CHAPTER XII

LOCAL AND DISCONTINUOUS ILLUMINATION

90. **Transitory Illumination.**—When transient illumination is very strong, as in the case of a single powerful electric spark, objects may apparently be seen even three or four times in succession. The retinal image is recurrent. McDougall has shown that, while with weak illumination one pulse only occurs, the number of pulses increases with the intensity. After the first strong pulsations cease, a fainter image, small at first, seems to grow to the size of the pulsating image and then gradually dies away. McDougall calls this the Secondary Image, the pulsating image being the Primary Image. The latter lasts during one-tenth of a second, and an interval of about one-fifth of a second separates it from the secondary image which lasts about one-third of a second. Other observers record a Tertiary Image, and Hamaker describes a Quaternary Image visible after illumination during one to four seconds.

The most suitable experimental mode of observation is by means of an illuminated radial slit in a rotating disc. The successive appearances are then visible in angular displacement, and an estimate of their durations is obtained in angular measure.

When the rotating slit is used the pulses of the primary image are seen as overlapping or separate bands having the colour of the objective light. McDougall found that, as the speed of the disc increases, faint bands appear in the interval between the primary and the secondary images. They are not seen if the eye be light adapted, and are also absent in foveal vision. Their general appearance is white with a bluish tinge; they are strongest with green illumination and are invisible with red illumination. Their characteristics are accordingly those of twilight or dark adapted vision.

McDougall regards them, and the subsequent secondary image, as pulses of the complete recurrent effect, and he considers their usual absence as due to "inhibition" (§ 93) by the primary image.

Opinion differs most with regard to the appearance of the secondary image. McDougall describes it as the ordinary positive after image, and Hess also describes it as having a faint tinge of the primary colour. This is possibly a dazzle tint, as it occurs when the intensity is sufficiently high. On the other hand, v. Kries says that it is complementary in colour to the primary image, but is absent in red light. Hamaker describes it as the best seen image for all colours of light when the light is stationary, and says that red and green illumination then give a strong complementary foveal image, while yellow and blue illumination give no foveal image. This indicates a condition of practically red-green vision, or yellow-blue blindness, for weak illumination of the fovea, in at least so far as the recurrent effects are concerned. In the case of the moving illumination, Hamaker describes the secondary image as complementary to the primary image, and as then showing no foveal effect. Charpentier says that it is violet in weak illumination and colourless in strong illumination.

The tertiary image is described by Hamaker as absent from the fovea both with stationary and with moving light, and as being little affected by dark adaptation. It has the colour of the primary image, and is better seen with red and yellow light than with green and blue. The latter feature indicates cone action, and suggests that absence of foveal action may be due to higher threshold values. The question of foveal action is on the whole left uncertain. Dark adaptation increases brightness but decreases colour, and this is most noticeable with red light.

The quaternary image is accompanied by a shortening of the tertiary image and requires fixed stimulation for a period of from one to four seconds. It is a true negative after image, complementary to the primary image, and surrounded by a halo.

The phenomena seen with moving transient light differ mainly from those seen with stationary light in that the summation of effects from precedent stimulations on the same

area of the retina are avoided. On the other hand, effects of adjacent stimulation may arise (§ 101).

On the whole the evidence indicates that the primary and tertiary images are positive, and that the secondary and quaternary images are negative or complementary. The positive tint sometimes seen in the secondary image is most probably the dazzle tint evoked by the primary stimulation. The fact that so much variation exists amongst the descriptions of the appearances is not a matter for surprise. For the phenomena must depend on the intensity, the wavelength, and the duration of the stimulus ; also on the extent of the retinal area stimulated, the conditions of fatigue, and the peculiarities of individual eyes. In its formal representation the whole phenomenon would be determined if the threshold values of the three fundamental sensations were known in terms of the variable conditions just specified.

The absence of the "interval bands" in red light ; and, ordinarily, of the secondary image when the test is made with the moving slit and red light ; is a result which may be predicable from the known high threshold value in red stimulation. And their absence in foveal vision is also predicable on the ground of high threshold values in colour stimulation of the cones. The great susceptibility of the cones to dark adaptation, i.e., to lowering of the threshold values for luminosity, is in consistence with the increase of brightness, of the secondary image by dark adaptation. The question of the explanation of its ultimate disappearance with extreme dark adaptation will be considered in the following section.

The absence of the positive tertiary image in foveal vision, which is asserted by Hamaker both in the case of the stationary and the moving image, might, if sustained, point to rod stimulation alone. And the result, also stated by him, that dark adaptation has little effect on its appearance, gives the same indication. But, on the contrary, the result, also given by him, that this image is better seen by red and yellow lights than by green and blue, points just as certainly to cone action as the source of the sensation. And, in fact, considerable difference of opinion exists on the question of the presence or absence of foveal action in this case. Dark adaptation, as already noted, is asserted to result in a brightening of the image

but in lessening of the colour, which is strongest with red light. An effect of this kind is at once explainable if, in consequence of dark adaptation, the threshold for the green and blue fundamentals falls below the stimulation value corresponding to the effect while the threshold for the red fundamental remains above it (Fig. 27, p. 181).

The characteristic phenomena cease when the duration of the pulse exceeds the action time (§ 91) by about two-thirds of its value; they begin to change when the excess is about one quarter. The action time for the faintest observable illumination is about one-fifth of a second. In other cases it may be three or four times smaller.

91. The Talbot-Plateau Law. The Action Time.—If the intensity of illumination incident on the retina varies in any definite manner with sufficient rapidity between two limits, the resultant impression is that of continuous illumination of a definite intensity. The law of the action, as determined experimentally by Talbot and Plateau, is that the resultant impression is the time average of the varying impression. Its accuracy has been verified to a high degree throughout a wide range. When the variation is periodic the result is expressed by the equation

$$\mathrm{IT} = \int i\,dt,$$

where T is the period and i is the instantaneous value of the stimulus of which I is the average value. The law asserts the equality of the steady stimulus to I.

When a single pulse of illumination lasting for a time T at intensity I is used, the total stimulus is IT. But the corresponding sensation requires time to rise, and also to fall to zero from its maximum value; so that, unless the law of rise and fall be known, no calculation of the corresponding pulse of sensation can be made. McDougall finds experimentally that the sensation is proportional conjointly to the steady intensity of the light employed and to the minimum interval of time for which it must last in order that the resultant intensity shall be equal to the steady intensity of the incident light itself.

Using McDougall's result and Fechner's law, we may write

$$kIT = \log\frac{I}{I_0},$$

where k is constant for any one wavelength and T is the minimum duration, called by McDougall the "Action Time," of the pulse which will give rise to the maximum sensation, while I_0 is the luminosity threshold for the light of steady intensity I. But the experimental results showed further that the expression on the left-hand side of the equation is correct for any time t less than T. So, if the other side be correct,

$$i = I_0 \varepsilon^{kIt}$$

is the law in accordance with which the effective stimulus varies as the development of the pulse proceeds. When t becomes equal to the action time, i takes the value I, and the formula ceases to apply when the exposure is longer.

92. **The Chromatic Action Time.**—The equation

$$kIT = \log\frac{I}{I_0}$$

being one asserting the equivalence of two sensations in respect of luminosity, and colour sensation values being expressible as white sensation values, while the luminosity threshold for all colours is identical in extreme dark adaptation, it is to be expected on theoretical grounds that the equation might apply equally to all colours at least at identical degrees of adaptation. In other words, x should be an absolute constant. The validity of the surmise is attested by experiments of McDougall on different colours. He found that different coloured lights had the same action time when the luminosity sensations were equal. But there is no a priori reason for the expectation that k will retain the same value when I_0 has different values.

Brückner and Kirsch investigated the minimum time for which a coloured light must act on the retina in order that colour should be perceived. They found that it is approximately proportional to the intensity of the sensation of the white light which is observed immediately before or immediately after the coloured light is employed, and that the effect is greater in the former case than in the latter. The law is thus of the same type as that given by McDougall.

Further, there is less difference of fatigue towards the coloured and the white lights when the latter produces its sensation first than when the white stimulation succeeds the coloured. Contrast is greater in the latter case. The more dissimilar the lights are, the greater is the influence of contrast (§ 108).

93. **Flicker Phenomena.**—When a disc exhibiting black and white sectors is set in rotation with increasing speed, the separate sectors, which are at first distinguishable, ultimately acquire the aspect of a single continuous grey disc. Intermediately there is an appearance of flicker which becomes less and less marked, until, just before complete fusion of effects, there is an aspect of peculiar sparkling brilliancy. This occurs at a little over eighteen alternations per second, which is close to the frequency of the normal action time. The same feeling of sparkle is associated with, say, the pink colour of a design of fine lines, alternately red and blue, when viewed from a distance at which the separate lines can just not be distinguished.

In the case of flicker the brightness seems greater than that of the final state. This may be due, as Brücke first suggested, to the superposition of the impression of an image of one bright sector upon the direct impression of another sector. But, if the action time falls short of the duration of the bright pulse at the speed of rotation which just destroys obvious flicker, the average sensation of brightness may, in accordance with the Talbot-Plateau law, be lessened by a further increase of speed to a value which makes these two times coincide. For the form of the law of the rise of sensation to its maximum and of the fall to its minimum is not considered in McDougall's expression for the intensity of the sensation due to a transient illumination. His statement perhaps implies that the form of the law of rise to the maximum is independent of the time of duration up to a limit equal to the action time. If the condition of fatigue alters appreciably during the course of illumination, that is, if the threshold value of the sensation so alters, the form of the curve of sensation may be such as to raise the average intensity at an intermediate speed over the high speed steady average. Strictly speaking, this includes Brücke's case.

The disappearance of sensation in recurrent vision may be accompanied by continuation of retinal stimulus provided that the threshold value of luminosity stimulation has risen to or above the effective stimulation at the time. So far as mathematical expression is concerned, an oscillatory variation in the threshold values could take complete account of recurrent phenomena. If, in this case, the primary source of the sensation is retinal, as it is in ordinary vision and possibly also in the self light, the source of "inhibition," i.e., of elevation of the threshold above the instantaneous stimulation, is necessarily cerebral, unless inhibition can arise in the retina, or the retino-cerebral connectors, apart from as well as because of the self light, whether perceptible or not. Increase of self light may be inhibitory to the effect of external stimulation: but, in any action after the external stimulation has ceased, the retinal cause of the self light is the efficient cause of sensation, and cannot lead to the self light inhibiting itself.

The self light must have oscillatory variation in recurrent vision if the cerebral or other inhibitory action is non-oscillatory: or the inhibitory action must be oscillatory if the self light decays steadily after suitable instantaneous retinal stimulation, which is perhaps to be expected on physical grounds.

In the practical application of flicker methods to chromatic photometry, the basis of one procedure is the postulate that two coloured lights are equally intense or luminous if, under otherwise identical conditions, the critical frequencies of interruption of each by a period of darkness are found to be the same. In the other and more usual procedure, it is postulated that the intensities of two mutually alternated illuminations are equal when no flicker is observed at a frequency of alternation characterized by the condition that flicker at once appears if the intensity of either illumination be altered. Experimental confirmation is given, as we have seen in § 59, by the coincidence of the results of the flicker method with those of the method of equality of brightness.

We have now to consider the laws regulating the flicker test and their theoretical relations.

94. **Porter's Laws and their Theoretical Deduction.**—

Using a rotating disc, half white and half black, in spectral illumination of definite wavelength, T. C. Porter found that the relation

$$n = k\log I + k',$$

where n is the critical frequency under intensity I and the quantities k and k' are constant, held throughout a range of intensity from one quarter of a metre-candle to 3,200 metre-candles. When the intensity was lowered below 0·25 metre-candle, the formula still held, but with a value of k about one half as great as previously.

He also showed that the time of maintenance of the sensation produced by a spectrum light in the close neighbourhood of its maximum is independent of the wavelength and varies only with the luminosity. This is of great importance since it exhibits a feature of the visual process which, like the action time, dominates all colour sensation.

Now, if the critical period of illumination be, as we have seen that it is, in the near neighbourhood of the action time, the illumination per pulse is $I\gamma$, where γ is the time of duration of the pulse. So, if there be n intermissions per second, since the sensation communicated per pulse is proportional to $I\gamma$ (§ 91), the sensation due to the continued external pulses will be proportional to $I\gamma n$ provided that the law of rise and fall of the consequent retinal pulsations be of such a nature that interference does not occur. McDougall's law of pulsatory sensation shows that the rise follows a simple law up to the action time, and the nature of the retinal effects of stimulation suggests an identical form of the law of decaying sensation when the stimulus ceases. But the Talbot-Plateau law implies that a definite sensation is produced by a definite pulse independently of the preceding pulses, so that $I\gamma$ has a constant value in the summed stimuli per second which produce the resultant sensation of intensity. Thus $I\gamma n$, being equated to $\log I/I_0$ gives

$$n = k\log\frac{I}{I_0},$$

which is precisely Porter's law if we identify k' with $k\log I_0$.

If, with the same ratio of the areas of the black and white sectors, the number of sectors in the disc be increased, it is

found that the speed of rotation must be increased if flicker is to remain absent. Since Porter's law

$$kn = \log \frac{I}{I_0}$$

gives $dI_0 = -kI_0 dn$ at the point of no flicker when I is constant, it follows that the required additional increase of n results from insufficient reduction of I_0 when the sector areas are reduced and their number is increased. This implies that, with increasing time of illumination, I_0 increases more rapidly at first than afterwards; which agrees with facts, including Porter's observation that the interval of time throughout which a pulse of sensation retains its maximum value, after cessation of the illumination, becomes larger through multiplication of sectors. It may be that the effect of the preceding pulse is prolonged to the next and constitutes self light stimulation which weakens the effect of the fresh pulse in accordance with Fechner's law. But the self illumination is added to the momentarily observed incident illumination; so that in this respect the result may be the same as if there were no continuation of the precedent pulse. On the other hand, the continued effect of that pulse includes a fatigue effect, which is added to the fatigue effect of the second pulse itself, and, with it, raises the threshold value and inhibits perception to some extent.

95. **Compound Intermittent Stimulation.**—When white light is intermitted colour effects will appear if differences dependent upon wavelength are manifest in the action time or the times of rise to, or fall from, the maximum of a pulse of sensation. Under certain conditions Helmholtz found that the preceding edge of a rotating white sector was tinged with red colour, and the following edge with blue. By the arrangement suitably of a black and white pattern on the disc of Benham's top, all types of colour may be seen. Fechner obtained yellow and blue.

The action and maximum-neighbourhood times being, as we have just seen, independent of colour, the decay times of sensation alone remain as the cause of variation.

96. **The Purkinje and the Reversed Purkinje Effects.**—We saw by Fig. 16, p. 111, that, in steady illumination, decrease

of intensity causes the maximum of brightness in the spectrum to change to a smaller wavelength. By the flicker methods it changes to a longer wavelength at very low intensities. That is to say, in flickering illumination the effect, known as the Purkinje effect, can be reversed.

Ives verified Porter's law, and extended it to the cases of different spectrum colours, suitably different values of the parameters k and k' being used for the different wavelengths. In each case the value of k is altered sharply to a smaller value when the illumination falls below about 0·25 metre-candle.

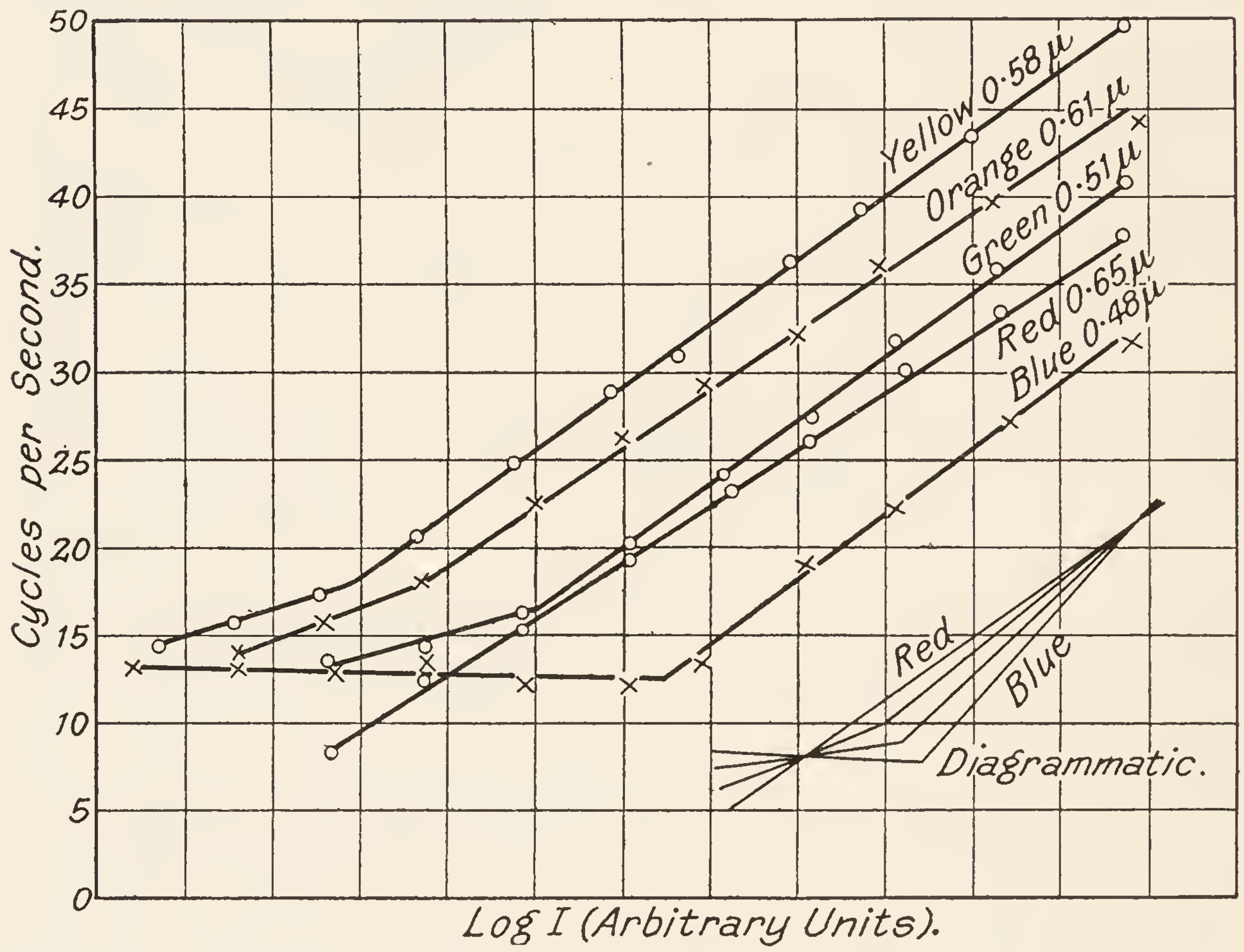

FIG. 25.—Curves of Critical Flicker Frequency and Intensity.

He connected the change of wavelength at the maximum brightness in the spectrum with the change in the value of k in the equation

$$n = k \log I + k'$$

by showing that the direct Purkinje effect and the reversed effect, respectively, occur at illuminations which exceed or fall short of the critical value of about one quarter of a metre-candle.

Because of the conditions noted in § 95, the explanation of the connection must be referred to a variation in the rate of

decay of sensation, which is dependent on the wavelength, or to a new action, or to both. Now we have found that the above law is deducible from the laws of intermittent illumination if we write $k' = k \log I_0$, where I_0 is the threshold value. Fig. 25 shows Ives' results, and the values of k and k' can be determined from the diagram for each wavelength both above and below the critical point. Hence the values of I_0 are determinable. They are shown in Fig. 26, the values of I_0 being plotted as ordinates against the wavelengths as abscissæ. Despite the apparent complication of the slopes of the lines in Fig. 25, the smoothness of the curves in Fig. 26 is remarkable.

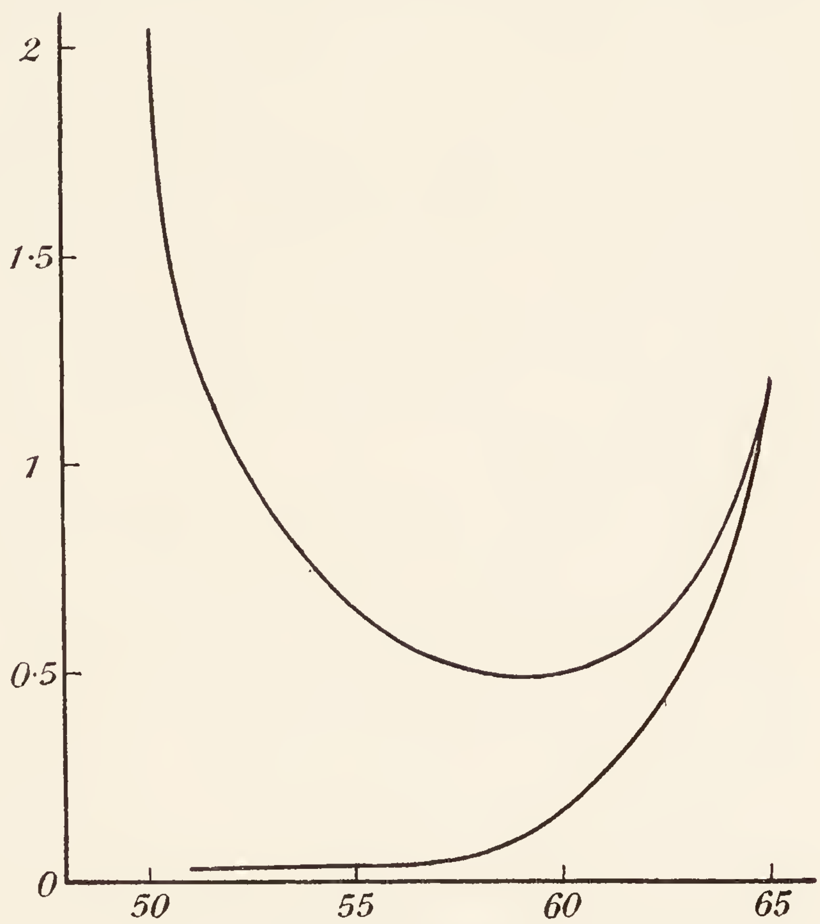

FIG. 26.—Threshold Values deduced from Ives' Curves.

In the equation at the commencement of § 92, k was regarded as constant in at least the case of constancy of the threshold value at extremely low intensity. But, under other conditions, it might rather be expected on physical grounds that the meaning of the equation would be that the sensation was proportional to the sensation value. On this view k would vary inversely as I_0. And the change in k for any colour in Fig. 25 should depend on the values of I_0 at the corresponding wavelength in Fig. 26. Thus the coincidence of the two branches of the curve of I at wavelength 65 expresses the absence of a change of slope in the line for that light in Fig. 25; and the increasing difference of ordinates with shortening wavelength in the former figure corresponds to the increasing change of slope at the critical points in the latter figure.

The changing values of both k and k' are thus explained in terms of the change in the threshold value I_0 alone (see further, Chapter XIV).

97. **Measures of Fatigue, Inhibition, and Adaptation in terms of the Threshold Values.**—In continuation of the remarks made at the end of § 94, it may be restated that Fechner has shown how the influence of self light may be included in the mathematical formulation dealing with intensity, and that Helmholtz has extended the formulation so as to include the case of the influence of self light upon colour, and that not merely in weak illumination, but even in the extreme case when dazzle occurs. The three fundamental threshold values r_0, g_0, b_0 are therefore free for the expression of the effects of the processes of fatigue or adaptation upon the sensation called forth by a given retinal illumination. They are functions of the time which prescribe the extent to which fatigue or inhibition has proceeded under light adaptation or the extent to which recovery from it has taken place under dark adaptation.

The threshold value I_0 for any light may therefore be used directly as the expression for the amount of fatigue. Since it expresses the magnitude of the illumination which must be exceeded before sensation ensues, it may also be used to express the partial amount of inhibition present. And its reciprocal can be taken as the measure of dark adaptation.

Fatigue, light adaptation, dark adaptation, and inhibition are thus brought within the range of quantitative numerical expression. The only other quantity which may require such inclusion is the maximum capacity for illumination corresponding to the threshold as the minimum.

98. **Areal Distribution of Flicker Effects.**—It has been found that the peripheral parts of the retina require, for continuous sensation, more rapid alternation of illumination than the central parts. But Ives showed that fatigue ensues with great rapidity on intermittent stimulation ; so that the sensitiveness to flicker soon becomes smaller than the central value.

Red light requires more rapid alternation at the fovea than blue ; and blue light requires greater rapidity at the circumference than at the centre. This is also in agreement with the general trend, exhibited in Fig. 26, of the distribution of the

threshold value throughout the spectrum according as the condition of fatigue corresponds to light adaptation or to dark adaptation. In the state of light adaptation I_0 is greater for blue light than for red, and conversely at low illumination.

The critical frequency of flicker depends, at a given intensity and threshold value, upon the colour of the light ; and, for a given wavelength, it depends on the intensity. Its discrimination takes cognizance both of colour variation and of change of intensity. Its results indicate the condition of fatigue in the three fundamental perceptions as being, along with the retinal stimulations, determinative of sensation in at least so far as self light in perhaps its extreme manifestation of dazzle does not take part.

Further consideration of the areal distribution of the effects of both steady and transient or intermittent illumination will be made in Chapter XIV.

CHAPTER XIII

AFTER IMAGES AND CONTRAST COLOURS

99. **Positive and Negative Images.**—The continuation of the subjective effect of external stimulation of the retina may give rise to a sensation of brightness or colour for some time after all direct excitation has ceased. Examples of this kind have already been discussed in connection with recurrent vision after brief stimuli. But prolongation of effects can take place, and can be manifested even more strongly, when the time of exposure of the retina to external excitation greatly exceeds the minimal action time. It is usual to refer to the two chief images, the first and the second alone; the first positive, that is, of like type with the luminous object, and the second negative, or of type complementary to that of the external source. But a series of images may be observed; and a descriptive statement of the appearances is further complicated by the facts that the images may be observed under the conditions of absence of further external action, or under stimulation of the retina by secondary external illumination, like or unlike the primary illumination, and of various intensities. The varying conditions of fatigue and adaptation, and even of inhibition, have profound influence.

100. **General Aspect of the Images. Blackness.**—The positive after image can readily be seen by fixing the line of vision in the direction of a bright light from which the eye is directly shielded, and then steadily and quickly, but without any jerk, removing and replacing the shield. The image dies away with more or less rapidity, and may be succeeded by a negative image.

If the light be white and has a dark background, the positive image is strong, and it is also strong with strong coloured light on a dark background. But, under the latter condition, the

negative image is very strong and may succeed the positive so rapidly, or be superposed upon it so fully, that the latter is not observable. McDougall has shown that, if the sharp boundary between the bright area and the dark background be shaded off, so that the bright and dark parts merge into each other gradually, the negative image does not interfere so powerfully and the positive image can readily be seen, though the effect is not so much obviated by this process when coloured light is used as when the light is white.

When the positive image is seen, it may at once be replaced by the negative image if the eye be directed towards a uniform and not too strongly luminous background. With a suitable illumination, the consequent negative image may be just sufficiently strong to annul the positive image, so that no image whatsoever is seen at the moment, though a sufficient difference in the rates of decay of the two might cause one to preponderate. Reappearance of an image may be due to more rapid decay of a stronger superposed one, or it may be a consequence of removal of inhibition.

In this way also we may see a coloured after image of a white object. When extremely intense illumination is used—for example, strong sunlight to which the eye is gradually adapted till all colour is removed from the landscape (which may be a dangerous experiment to carry out)—after images may continue long, visible in full daylight, and vanishing only after weeks have lapsed.

The production of coloured after images may take place even when the light of which they are the images appears to be uncoloured to the eye concerned. This was very specially noticeable in the case of colour blindness referred to in § 115. Green light, matchable with grey, gave a strong red after image. Red light, on the other hand, gave no after image; nor was any seen with yellow or blue lights, each of which was also matchable with grey (*Trans. R.S.E.*, 1896).

If the line of vision be fixed, an ordinary negative after image may disappear and reappear successively; and these reappearances may be alternated with the appearance of a positive image.

All after images are, strictly speaking, manifestations of the so-called self light of the eye raised by precedent conditions to

an intensity sufficient for direct sensation under circumstances in which it would normally be quite invisible. The sensation of blackness is certainly, in the psychological sense, a direct sensation. But, from a black body, no physical stimulus can reach the retina, and from an unstimulated retina, no physiological stimulus can reach the brain except in so far as there may be retinal self changes of the same type as those induced by external illumination. This action is the action which originates the self light in so far as it possesses a retinal origin. Its seat is the seat of transformation of the energy of light into the energy which directly causes physiological action. In so far as any other source of sensation may exist, its seat must be the seat of transformation of the energy of physiological action into the energy involved in the production of sensation, whatever the nature of that process may be.

A cerebral localization of the latter kind, to which general opinion seems to incline, would be indistinguishable from a retinal localization. It is the existence of this self light, whatever be its origin, which gives rise to the idea that there can be degrees of blackness. If we choose to regard the background of vision of the rested and non-externally illuminated eye as black, we can undoubtedly, by fatigue of a portion of the retina (or corresponding portion of the brain) through looking at a white object, see, on closing the eye, a negative after image of the object which appears to be blacker than the general background. But we ought not to conclude that there can be a black which is "blacker than black." The error lies in calling the normal background black when external light is excluded.

McDougall's elaborate experimental and critical investigation (*Mind*, X, N.S., 1901) of the phenomena of colour vision in connection with the trichromatic theory is of great value. He concludes that three main colours, red, green, and blue, occur in superposition or successively, in the development of after images; and that their manifestations are more evident when their inducing illumination is strong. Between two periods of this strong colour impression, there occurs a period of fluctuation and struggle of the sensations. He finds that fatigue or adaptation, by a strong colour of one of these three types, is followed in subsequent darkness by a positive after

image, which lasts for a considerable time, and then by a strongly complementary negative after image; and that, in weaker illumination, the positive image does not appear. Details of the phenomena vary very greatly under different conditions, even to inversion of order in time. With too long (about two minutes) exposure to the direct light, the effects are weakened.

When a sharply contrasting bright object is viewed against a dark background, the negative after image seen when the eye is directed towards a moderately bright white background is bordered by a bright halo. This disappears when, as mentioned above, the object is shaded gradually into the background. In the case of a coloured object the halo is of like colour when it is visible, but, as stated above, it is the superposed or quickly following complementary colour of the negative image which is more usually observed.

101. **The Influence of Self Light, Fatigue, and Defatigue.**—The strength of the halo in the complementary image is due to the lowering of the threshold value by the absence of precedent illumination on the part of the retina where it appears. By McDougall's method of a graduated demarkation between bright and dark parts of the inducing field of vision, the condition of fatigue varies continuously across the boundary. Therefore the condition of stimulation by the subsequent uniform field varies continuously, and so no sharp halo appears.

In the phenomena of after images we come into direct contact with a new influence of retinal stimulation which we have not hitherto taken into account. The question involved is that of the dependence or independence of the effects of stimulation of one part of the retina with respect to separate stimulation of another part, either previously or simultaneously. It is indeed well known that an inter-relation exists between the effects of independent areal stimulations. Thus McDougall has shown that perception of an image thrown upon one part of the retina may be entirely inhibited by the incidence of fresh illumination on another part.

The phenomena of after images compel the recognition of the defatiguing influence of a definite stimulation of one retinal region upon adjacent parts. Whether that influence

is developed in the retina itself, during propagation of action along the retino-cerebral connecting pathways, or in the brain itself, is a problem which is certainly of great interest to the physicist. But it is not his direct concern. The action is, in any event, expressible as to its formal relationships by means of the component fundamental threshold values. These have to be regarded as dependent, in time and space alike, upon adjacent illumination as well as upon the direct illumination.

In a non-uniformly illuminated white field, defatigue of the darker parts may brighten these in spite of their own illumination which produces fatigue, and fatigue of the bright parts by their own illumination may be more effective than their defatigue by the adjacent less bright illumination. In this case the tendency would be for the illumination to become gradually more uniform in aspect, and this effect is well known. If one directs the gaze fixedly towards a point in a shaded part of a white ceiling, the shaded parts will seem to brighten appreciably by slow degrees, the contrast becomes less between the shaded part and its brighter surroundings, and at last all difference disappears and only a uniform field is seen. But the illusion is destroyed at once by even a slight motion of the line of vision, as it should be.

In a uniformly illuminated white field, fatigue of retinal parts by the incident light exceeds defatigue by adjacent stimulation, and the consequent increase of the threshold value gives rise, on cessation of the external stimulation, to the temporarily existent positive image. The increase of the threshold value has then a positive magnitude as self light, evident during the absence of any or too strong external illumination, and which may inhibit that external illumination from perception if it be sufficiently weak, as when an eye, dazzled by strong lights, cannot perceive even bright objects through the consequent glare of the intensified self light. But the increase in the threshold value may also have inhibitory effect apart from evident self light.

In the case of coloured illumination affecting mainly one fundamental sensation, there will correspondingly be defatigue of the other two arising in the same retinal region as well as outside it. Thus the positive image corresponding to the incident colour tends, in consequence of the development of

fatigue, to be eclipsed by the action of subsequent illumination upon the other fundamentals, reinforced as it is by defatigue. This effect will not be so strong in the case of coloured light which contains a considerable amount of white, and so affects the three fundamentals more equally. Hence it is to be expected that yellow and blue lights should have a less strongly marked after effect than red and green; which is in fact observed.

The characteristics of the halo formed round the negative after image of a coloured light give further information regarding the processes of fatigue and defatigue. Its quality is positive if the light be neither too strong nor be regarded for too long a time. Continuation of the objective stimulus causes the positive colour to fade gradually until the halo becomes colourless and finally reverses to the complementary colour while the image of the light itself is positive. The general tendency is for the image and its halo to be complementary in colour—which may be the direct result of defatigue of an adjacent area. The complementary effect grows continuously in importance. It becomes equal to the direct positive effect when the halo becomes white, and afterwards exceeds it. At first the defatigue of retinal centres adjacent to the illuminated area, and of the same type as those which are strongly stimulated in that area, is greater than the defatigue of centres of different type. Its rate of increase is initially greater, but finally becomes less than that of the complementary type. The halo is strong if there is defatigue of all centres. The illuminated area is heavily fatigued and therefore cannot give strong expression to the intensified self light which constitutes the after image. Thus the halo may seem to be far brighter than the image which it surrounds. This occurs readily, and even complete inhibition of the image may take place. This happens when the fatigue is so great that the threshold value is equal to or exceeds the intensity of the self light, that is, of the continued stimulus, whether of retinal or other origin, which is transmitted, for transformation, to the psychical centre, but which is inhibited from transformation in consequence of the conditions subsisting at the seat of fatigue.

Herein it is essential to remember that the magnitude of a sensation is as dependent upon the threshold value as upon the

actual value of the illumination. The sensation value, being the ratio of the actual value of the illumination to its threshold value, may, in feeble illumination, be very great if the threshold value be low ; and conversely. It is customary to speak of the self light observed in total darkness as being very weak, but the sensation may be occasionally, during the irregular fluctuations of the self light, so strong as to give rise to the same feeling of pain as that which is produced by an intense light before fatigue develops.

If there be the same mutuality of relationship between fatigue and defatigue as that which characterizes physical actions which can be maintained in a state of kinetic equilibrium, defatigue at one region may induce fatigue at a neighbouring region. In that event the defatigue located at the halo of an after image may automatically tend to aid, or to oppose decay of, the fatigue directly induced by the incident light at the locus of the image.

102. **Influence on Newton's Law.**—Since no definite observations on colour can be made except under conditions of more or less approximate constancy of fatigue, self light, adaptation, or defatigue, it becomes a matter of great importance to ascertain in how far variation of these conditions may affect the accuracy of Newton's law of colour mixture. Thus if a colour equation be formed between a yellow light and a mixture of red and green lights with the eye in a state of fair absence of fatigue, it may occur that an otherwise identical test, differing only in that the eye is strongly fatigued to the yellow light, shall show that the red and green lights do not exhibit proportionate fatigue. In that case the match must cease to hold.

As a matter of fact it is found that, so long as the lights fall upon the fovea, a match made before fatigue remains accurate after fatigue. If, in the preceding case, the fatigue be induced by yellow light, both the monochromatic yellow and the compound yellow become whiter to an equal extent ; if the fatiguing light be blue, both become more saturated. This shows, in the first place, that both yellows contain white. For, if the red and green perceptive centres alone were stimulated by them, fatigue by the yellow could only weaken the intensity of each equally if no change of colour took place ; and fatigue

by blue could only intensify both. The decrease of saturation which occurs after fatigue by yellow light shows that defatigue of the blue perceptive centre has taken place ; the increase of saturation which follows fatigue by blue light shows that fatigue, being due to perception by blue centres except in so far as that blue light also affects the red and green centres to some extent, is accompanied by defatigue of the red and green perceptive centres. The process of fatigue is a compound process, always involving fatigue on the whole, in which partial processes of defatigue may take place.

The gradual loss of colour of a strongly coloured object which is very fixedly regarded for a very long time occurs in accordance with this law of combined fatigue and defatigue. A perceptive centre which is stimulated in excess, that is to say, one which contributes mainly to the resultant colour impression, is directly fatigued on the whole ; for any tendency to defatigue in virtue of the action of the light upon the other centre or centres is weaker. A feebly stimulated centre, on the other hand, undergoing correspondingly slight direct fatigue, may be defatigued in excess by the action of the light upon the other centres. So the tendency in that case is always towards equalization of the three sensation values, the threshold value of a strongly stimulated centre rising while that of a weakly stimulated centre falls. The case of a strongly coloured red surface whose image extends over the greater part of the retina is specially striking. The colour fades very slowly but steadily, giving rise to a peculiar sensation of loss of power of the eye, until the field becomes quite grey and colourless. At this stage the state is one of kinetic equilibrium, and a subjective fluctuation, like those which are always taking place in the self light, may suddenly result in a rise of the threshold value of the strong red stimulation or a fall in the threshold values of the weak green and blue stimulations. If that occurs the colour of the field will suddenly flash into perception as a strong green. The effect is unusually striking, but is not easy to attain.

It is obvious that fatigue must be accompanied by a diminution in differential sensitivity as exhibited in the corresponding sensation curves. Thus the range of error possible in establishing a given match is increased through fatigue by an

independent illumination. This was pointed out by Watson. It follows that a match made after fatigue may not continue to be a match after recovery from the fatigue. But a match made under conditions of greatest sensitivity will continue to be a match after fatigue. The more stringent are the conditions of sensitivity, the more rigidly is the law of Newton obeyed.

103. **Theoretical Deduction of Fatigue Effects: (1) Steady Decay, (2) Change in Saturation.**—As the simplest assumption which can be made towards a formulation of the processes of fatigue and defatigue preliminary to experimental test, we may postulate that fatigue of a centre occurs at a rate which is proportional to the sensation value instantaneously associated with its action; and that defatigue of that centre goes on at a rate which is proportional to the sum of the sensation values which are, at the same instant, effective at the other two centres. These assumptions can easily be generalized, but the simplest case is sufficient for at least a general description of the observed phenomena.

If r/r_0, g/g_0, b/b_0 represent the fundamental sensation values, the postulates give

$$\frac{d}{dt}\left(\frac{r}{r_0}\right)=-\varrho\frac{r}{r_0}+\lambda\left(\frac{g}{g_0}+\frac{b}{b_0}\right),$$

$$\frac{d}{dt}\left(\frac{g}{g_0}\right)=-\varrho\frac{g}{g_0}+\lambda\left(\frac{b}{b_0}+\frac{r}{r_0}\right),$$

$$\frac{d}{dt}\left(\frac{b}{b_0}\right)=-\varrho\frac{b}{b_0}+\lambda\left(\frac{r}{r_0}+\frac{g}{g_0}\right),$$

where ϱ and λ are regarded as constants, and we presume for simplicity that λ has the same value for the green and the blue stimulations. The solution of these equations gives

$$\frac{r}{r_0}=\mathrm{B}_r\varepsilon^{-(\rho+\lambda)t}+\frac{\mathrm{A}}{3}\varepsilon^{-(\rho-2\lambda)t},$$

$$\frac{g}{g_0}=\mathrm{B}_g\varepsilon^{-(\rho+\lambda)t}+\frac{\mathrm{A}}{3}\varepsilon^{-(\rho-2\lambda)t},$$

$$\frac{b}{b_0}=\mathrm{B}_b\varepsilon^{-(\rho+\lambda)t}+\frac{\mathrm{A}}{3}\varepsilon^{-(\rho-2\lambda)t}.$$

Here A is the sum of the initial sensation values, and the three other constants are subject to the condition

$$B_r + B_g + B_b = 0.$$

The quantities r, g, b are to be taken as constants. They are the components of light which illuminates an eye, whose fundamental perceptivities, given by the above equations, had been brought by precedent illumination to the values

$$\frac{1}{{}_1r_0} = \frac{A+3B_r}{3r}, \quad \frac{1}{{}_1g_0} = \frac{A+3B_g}{3g}, \quad \frac{1}{{}_1b_0} = \frac{A+3B_b}{3b}.$$

Each B is the deviation of the corresponding initial sensation value from the mean of the three initial sensation values. The equations show that ϱ cannot fall short of 2λ. And the rate of change of any sensation will be negative or positive according as the value of the corresponding B, necessarily negative, is of magnitude exceeding, or falling short of, the value $\frac{A}{3}(\varrho - 2\lambda)/(\varrho + \lambda)$. This is the condition which was postulated in § 101 with respect to the phenomenon of the vanishing of differences in a steadily regarded, non-uniformly illuminated white field.

It is not possible to take account, in these equations, of any uncontrollable fluctuations of the self light. Therefore, even if vindicated in other respects, they can only be applied when the illumination is so strong as to make the self light steady or to overcome the influence of its variability. Therefore, also, they may not be usefully applied to illumination of the retinal rods in so far as with them greater uncontrollable variation is coupled.

Further, there is no explicit account taken in the equations of any decay of fatigue in the absence of external illumination. But the form of the equations is independent of the intensity of the external illumination. Thus they may apply to the externally unstimulated variations if r, g, b be taken as the components of the self light. In that event these quantities would be regardable in general as the components of the total equivalent stimulation, external and internal. For it is not possible to discriminate in sensation between the effects of external and internal origin. This is recognized mathe-

matically in the double variability of the sensation value through both numerator and denominator.

It is to be noted that dissymmetry comes in through the quantities B alone. Should ϱ be very large in comparison with λ, decay follows the simple exponential law without change of colour. Should it be equal to 2λ, there is simple exponential decay with increasing whiteness. The first exponential is the more powerful one, therefore colour always tends to vanish ultimately in this special case.

104. **Further Applications : (3) Inhibition, (4) Change of Colour, (5) Difference Extinction, (6) Change from Positive to Negative.**—If one portion of the retina be stimulated by light whose components are r, g, b ; and if another portion be simultaneously under like conditions with values r', g', b', r'_0, g'_0, b'_0, the three equations at the commencement of last section require the addition of terms such as

$$m'\frac{r'}{r'_0}-n'\left(\frac{g'}{g'_0}+\frac{b'}{b'_0}\right),$$

and similar equations have to be written down for the other area. Thus we have to deal with a system of six linear differential equations of the first order in six variables. The quantities m', n' depend on the size and locality of the second area.

Inhibition.—A simple case arises if the two lights are identical in wavelength. To find the general nature of the action it is sufficient to presume that only one type of centre is affected, say the red. The two requisite equations are then

$$\frac{dx}{dt}=-\varrho x+my,$$

$$\frac{dy}{dt}=-\varrho' y+m'x,$$

if we write for shortness x and y instead of r/r_0 and r'/r'_0. The solution is

$$(\lambda_1-\lambda_2)x=(x_0+\lambda_2 y_0)\lambda_1\varepsilon^{-(\rho-\lambda_1 m')t}-(x_0+\lambda_1 y_0)\lambda_2\varepsilon^{-(\rho-\lambda_2 m')t}$$

$$(\lambda_1-\lambda_2)y=(x_0+\lambda_1 y_0)\varepsilon^{-(\rho-\lambda_2 m')t}-(x_0+\lambda_2 y_0)\varepsilon^{-(\rho-\lambda_1 m')t},$$

where x_0, y_0 are the initial values and λ_1, λ_2 are the roots of the equation

$$\lambda^2 m'-\lambda(\varrho-\varrho')=m,$$

λ_1 being the algebraically larger root. The sensation value x can only vanish at a finite time given by the condition

$$\log\frac{(x_0+\lambda_1 y_0)\lambda_2}{(x_0+\lambda_2 y_0)\lambda_1}=-m'(\lambda_1-\lambda_2)t.$$

But, x_0 being necessarily positive, in fact greater than unity, we have

$$(x_0+\lambda_1 y_0)\lambda_2<(x_0+\lambda_2 y_0)\lambda_1,$$

so that m' must be positive. If the two quantities in brackets here have opposite signs, y cannot vanish in finite time. Therefore the image which corresponds to x is inhibited before t in this case. Again y also can vanish at time t' given by

$$\log\frac{(x_0+\lambda_1 y_0)}{(x_0+\lambda_2 y_0)}=-m'(\lambda_1-\lambda_2)t'.$$

With m' positive, t' will be negative if the logarithm exceeds unity, which occurs if both λ_1 and λ_2 are positive. But the equation for λ,

$$\lambda=\frac{\varrho-\varrho'}{2m'}\pm\sqrt{\left(\frac{\varrho-\varrho'}{2m'}\right)^2+\frac{m}{m'}},$$

with m' positive, then requires $\varrho>\varrho'$ and m negative. Also, for reality $(\varrho-\varrho')^2>4mm'$. Under these conditions x is inhibited and y decays exponentially.

If λ_1 and λ_2 are both negative, the logarithm is negative, so that t' also is positive. And now it is necessarily greater than t since λ_2 is numerically greater than λ_1. Here it is most important to note that, although on the Young-Helmholtz theory the sensation $\log y$ cannot take negative values, there is no a priori reason why the sensation value, though then ineffective in the production of sensation, should not take values less than unity or even negative. The question is one for observation to settle.

Thus we get inhibition of x and exponential decay of y with m' positive, m negative, and $\varrho>\varrho'$; and also in any case with m' positive provided that λ_1 and λ_2 respectively exceed and fall short of the value $-x_0/y_0$ algebraically, which may occur with m negative and $\varrho>\varrho'$ or with m positive and $\varrho'>\varrho$. The stronger image inhibits the weaker. And if the one be sufficiently weak

relatively to the other, the inhibition may practically occur instantaneously.

This is the case in the sensory estimate of the blackness of the highly luminous sun-spots.

The term stronger image here does not refer to the intensity of the incident light, but to the intensity of the sensation produced by it.

Colour Change.—In the above investigation y may evidently represent a stimulus affecting a different type of centre; say green instead of red. So that the result is independent of the nature of the two stimuli, and indicates colour change when the stimuli are of different type. We may add to the expressions for dx/dt and dy/dt a term proportional to a third sensation value z which may still let x vanish in finite time while y does not do so. Nine disposable constants are now brought in instead of four as above, so that the apparent change of colour which light of any kind may undergo during, and in consequence of its continued incidence upon the retina is readily accountable for on the trichromatic theory.

Extinction of Field Intensity Differences.—This phenomenon, one example of which has already been considered in § 101, necessarily results if one sensation value, x say, is increasing, while the other y is decreasing. Thus the effect follows if, with

$$\dot{x} = -\varrho x + my,$$

$$\dot{y} = -\varrho' y + m'x,$$

we have

$$\frac{y}{x} > \frac{\varrho}{m}$$

and also

$$\frac{y}{x} > \frac{m'}{\varrho'},$$

provided that m' be positive. The only necessary condition for uniformity of sensation, in addition to the presumption $x_0 < y_0$, is that, at some finite time t, we shall have $x = y$ which gives

$$\log \frac{(x_0 + \lambda_1 y_0)(1 + \lambda_2)}{(x_0 + \lambda_2 y_0)(1 + \lambda_1)} = (\lambda_1 - \lambda_2) m' t.$$

A numerical illustration, with m' negative, in which x increases with time initially, is given in Fig. 27, the selected values being

$m=2$, $m'=-1$, $\varrho=0$, $\varrho'=3$, $x_0=10$, $y_0=20$. In that case inhibition of one image occurs early.

Change from Positive to Negative.—If the time exceed the value given by the preceding equation while x and y are still positive, the relative magnitudes of x and y are reversed. The brighter image becomes the weaker, and conversely; and if colour be present, the x type is replaced by the y type, or conversely. Whether or not the retinal or cerebral conditions are such as to render the effect possible is a matter for observation to settle.

When the stimulus comes from the effective self light, the phenomenon is that of the well-known replacement of the positive after image by the negative after image.

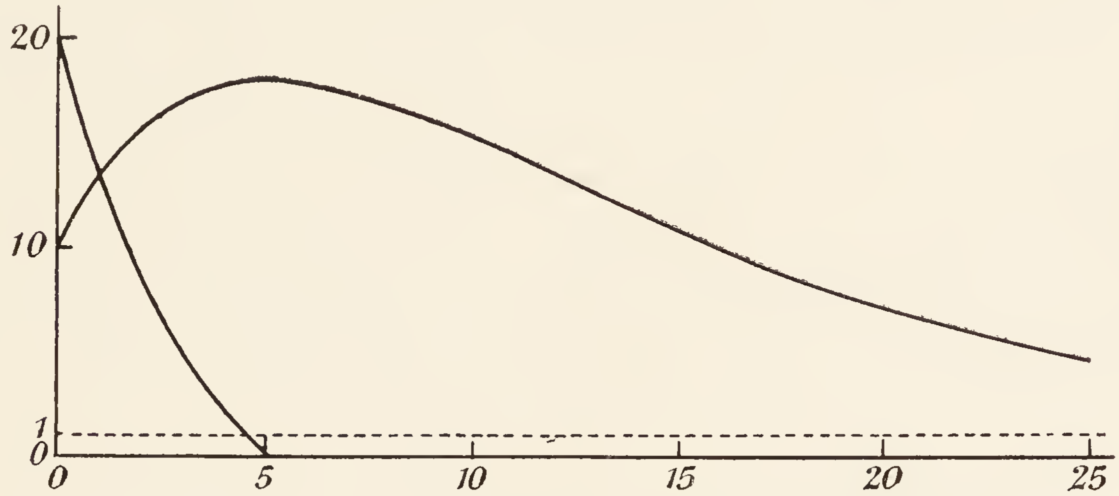

FIG. 27.—Curves of Fatigued and Inhibited Sensation.

105. **The Origin of Colour Blindness.**—If, for the sake of simplicity of treatment, we take instead of the three general expressions for the rates of variation of x, y, and z, the case in which

$$\dot{x}=-\varrho_1 x+m(y+z),$$
$$\dot{y}=-\varrho_2 y+m(z+x),$$
$$\dot{z}=-\varrho_3 z+m(x+y),$$

we have

$$\frac{d}{dt}(x-y)=-(\varrho_1+m)(x-y).$$

Hence

$$x=x_0\varepsilon^{-(\rho_1+m)t}, \quad y=y_0\varepsilon^{-(\rho_1+m)t},$$

the ratio of y to x remaining always at the constant magnitude

which it originally possessed. By the help of these equations we find

$$z=\frac{m(x_0+y_0)}{\varrho_3-\varrho_1-m}\varepsilon^{-(\rho_1+m)t}+\left(z_0-\frac{m(x_0+y_0)}{\varrho_3-\varrho_1-m}\right)\varepsilon^{-\rho_3 t}.$$

This expression vanishes at a time t for which

$$\log\frac{m(x_0+y_0)}{z_0(\varrho_3-\varrho_1-m)-m(x_0+y_0)}=-(\varrho_3-\varrho_1-m)t.$$

With $\varrho_3>\varrho_1+m$, t will be positive and finite if

$$z_0(\varrho_3-\varrho_1-m)>2m(x_0+y_0).$$

Also t can be excessively small if the inequality be very slight. At an earlier time still, z becomes unity and the sensation vanishes. Dichromasy passes into monochromasy.

If the colour perception of an eye were regulated in accordance with this scheme, the perception would be at best dichromatic. This suggests that the origin of dichromasy and monochromasy may lie simply in practically permanent inhibition of a particular colour as a result of the existence of definite cross connections between the fundamental stimuli, and not as the result of intrinsic functional defect. The inhibition of z is complete throughout finite time in the above special illustration for any colour whatever, for x_0, y_0, and z_0 may have arbitrary values.

It is of course the excess of each of the sensation values, x, y, z, over unity that corresponds to the usually employed x, y, z of Lambert's colour pyramid.

For inhibition of z throughout all appreciably finite time, it is only necessary that ϱ_3 should be sufficiently large. And there is no necessary limitation of z to a fundamental sensation. In the more general event the vanishing of $z-1$ means the vanishing of a homogeneous linear function of the three fundamental magnitudes $\bar{x}-1$, $\bar{y}-1$, $\bar{z}-1$, which are Helmholtz's x, y, and z respectively, if variation of the threshold values be disregarded. Thus Helmholtz's treatment of the phenomenon of colour blindness follows directly.

106. **Recurrent Images.**—In the preceding treatment it has been tacitly assumed that the exponentials in the expressions for x, y, and z are real. But there is no necessary limitation of this kind. The roots of the quadratic for λ

(§ 104) become imaginary if m and m' be of opposite sign while the numerical ratio of m to m' exceeds the value of the square of $(\varrho-\varrho')/2m'$. In that event the law of decay of the sensation value is simple harmonic with exponentially decreasing amplitude. Whether or not such oscillations of sensation take place under external stimulus is a matter for observation to settle, and their occurrence in the case of after images is well known. Phenomena of this type are of great complexity, for causes of variation in the results arising from variations of fatigue and defatigue or adaptation and from the nature and intensity of the precedent and existent illuminations are constantly present. There lies open a wide field for further investigations to systematize knowledge of the subject. But the simplest theoretical assumptions may be used as a systematic guide.

We may consider afresh, in this connection, the system of two interconnected equations

$$\dot{x} = -\varrho x + my,$$
$$\dot{y} = -\varrho' y + m'x.$$

The condition for imaginary roots of the quadratic in λ is that m and m' shall have opposite signs, with the further provision

$$(\varrho-\varrho')^2 < 4mm'.$$

If we write for shortness

$$2\beta = \sqrt{-(\varrho-\varrho')^2-4mm'}$$

the periodic solution is, with suitable choice of the time scale,

$$y = y_0 \varepsilon^{\frac{-\rho+\rho' t}{2}} \cos\beta t,$$

$$x = x_0 \sec a\, \varepsilon^{\frac{-\rho+\rho' t}{2}} \cos(\beta t - a),$$

where

$$\cos a = \frac{\varrho-\varrho'}{\sqrt{-mm'}}.$$

It appears, therefore, that oscillations occur which are of the same period in both sensations; that the period is greater the greater is the difference between the coefficients of direct fatigue; and that it is less the greater is the value of each cross coefficient whether it be that of fatigue or of defatigue. Further, if a be zero, there is no difference of phase, and the successive images exhibit no change of colour while their

amplitudes gradually die down. Again, if a be equal to π, the images are in opposite phases, that is, the colours alternate in the successive images without any overlapping.

It must be carefully borne in mind that these oscillations are oscillations of the sensation values, not of the sensations. No negative magnitude for the latter can occur according to the Young-Helmholtz theory. But it does not follow that the cause of sensation value cannot oscillate. So far as theory goes, sudden diminution of the threshold value by other action might reveal its latent power. An analogy may be found in the unidirectional current passing through the telephone of a wireless circuit which receives oscillatory electro-magnetic impulses.

When the general case of stimulation of the three fundamentals is considered, the solution of the equations

$$\dot{x} = -\varrho_1 x + m_1 y + n_1 z,$$

$$\dot{y} = -\varrho_2 y + m_2 z + n_2 x,$$

$$\dot{z} = -\varrho_3 z + m_3 x + n_3 y,$$

is of the form

$$x = \mathrm{A}\varepsilon^{-pt} + \mathrm{B}\varepsilon^{-qt}\cos\beta t,$$

$$y = k_1 \mathrm{A}\varepsilon^{-pt} + k_2 \mathrm{B}\varepsilon^{-qt}\cos(\beta t + a),$$

$$z = k'_1 \mathrm{A}\varepsilon^{-pt} + k'_2 \mathrm{B}\varepsilon^{-qt}\cos(\beta t + a'),$$

the epoch being suitably chosen. The appearance indicated by the three first terms on the right-hand side of the equations is that of a definitely coloured light decaying exponentially. The remaining terms show the superposition upon it of oscillations, which have the same period in each fundamental sensation, but which have in general different amplitudes and different phases in each. The amplitudes of these oscillations also decay exponentially at a common rate which is in general different from that of the non-oscillatory component.

In Fig. 28 the curves represented are

$$x = 12{\cdot}5\varepsilon^{-2t} + 12{\cdot}5\varepsilon^{-t}\cos 2\pi t + 12{\cdot}5\varepsilon^{-t}\sin 2\pi t,$$

$$y = 6{\cdot}25\varepsilon^{-2t} + 18{\cdot}75\varepsilon^{-t}\cos 2\pi t - 6{\cdot}25\varepsilon^{-t}\sin 2\pi t,$$

$$z = 6{\cdot}25\varepsilon^{-2t} + 18{\cdot}75\varepsilon^{-t}\cos 2\pi t - 3{\cdot}125\varepsilon^{-t}\sin 2\pi t.$$

The y and z curves differ little. If their average be considered, the diagram illustrates Hamaker's succession of primary, secondary, tertiary, and quaternary recurrent images (§ 90),

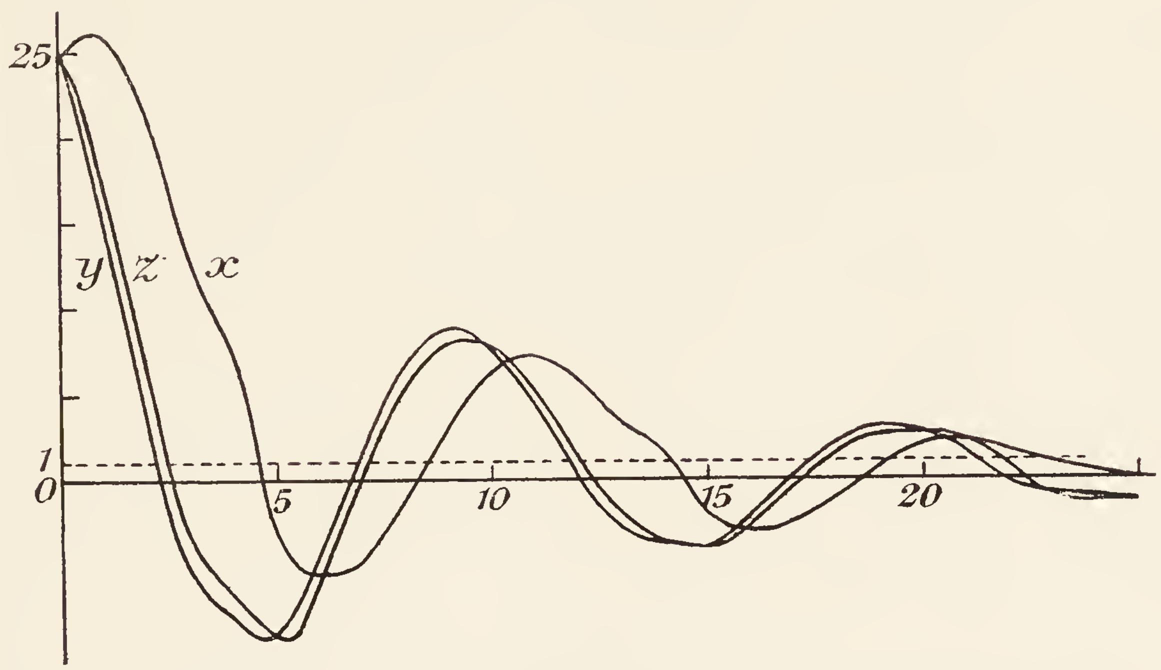

Fig. 28.—Oscillating After Images with Initial White Sensation.

alternately positive and complementary, provided that the numerical values of the ordinates be reduced by one half.

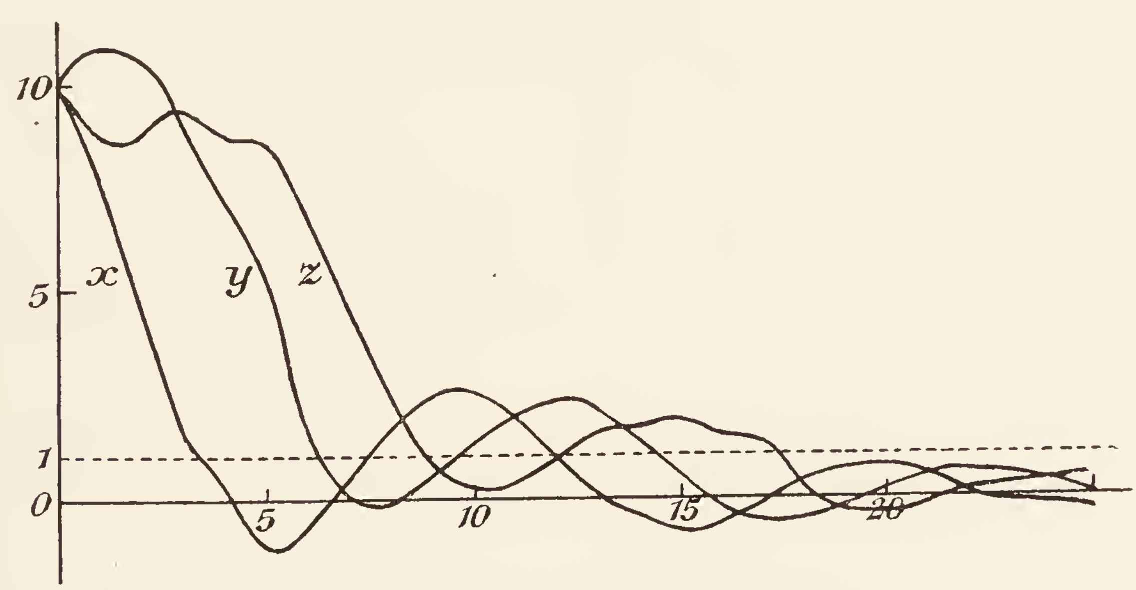

Fig. 29.—Oscillating After Images with Initial White Sensation.

In Fig. 29 the curves are

$$x = 5\varepsilon^{-2t} + 5\varepsilon^{-t}\cos 2\pi t,$$

$$y = 10\varepsilon^{-2t} + 5\varepsilon^{-t}\sin 2\pi t,$$

$$z = 15\varepsilon^{-2t} - 5\varepsilon^{-t}\cos 2\pi t.$$

The curves in Fig. 30 show ordinates in proportion to

$$x= \varepsilon^{-t}+\varepsilon^{-\cdot 3t}\cos 2\pi t,$$

$$y=2\varepsilon^{-t}+\varepsilon^{-\cdot 3t}\sin 2\pi t,$$

$$z=3\varepsilon^{-t}-\varepsilon^{-\cdot 3t}\cos 2\pi t.$$

If the upper dotted line corresponds to unit sensation value, there is a dark interval between the initial effect and the first triple set, beyond which inhibition takes place. The dark interval disappears if the lower dotted line corresponds to unit sensation value. In the former case the succession of colours, if x, y, z represent respectively blue, green, and red, is like that described by McDougall at the second stage of intensity (§ 107). Fig. 29 illustrates this also. The equations show that the triple set recurs as McDougall observed if the threshold value be low enough. The coefficients in the equations depend on the intensity and the conditions of fatigue, etc.

Inhibition of any of the components takes place when the corresponding sensation value falls to unity (Fig. 27). It is at first periodic, but the proportion of the visible interval to the invisible interval gradually decreases, and finally the inhibition becomes total. Sudden alterations of conditions, however, may cause the reappearance of the image through change of stimulation, either direct or indirect, or through similar change of the threshold value.

It should be noted that a sum of multiples of x, y, z as above expressed has exactly the same mathematical form, so that the results are independent of the particular set of fundamentals chosen.

107. **Experimental Confirmation.**—These results apply alike to the variations of direct and after images. As we have already seen (§ 90), oscillations of this general nature are well known. In the case of direct images the great difficulty in following the course of the time variations lies in the liability to a change in direction of the line of vision.

If one gazes steadily at a dark beam crossing a white ceiling in strong daylight, the gradual brightening of the beam is very apparent. A slight displacement of the line of vision makes the reality of the brightening evident. The appearance is that of a white haze covering the beam, and the bright sur-

roundings are correspondingly darker than at first. When all differences seemingly vanish, the impression is that of a haze or dazzle wiping out the whole background, just as dazzle produced by daylight prevents recognition of fairly bright objects in a dark room. But the conditions of direct illumination of this kind, of different intensities on different parts of the retina, are quite different from those which follow when the eyes are closed afterwards. Since the nine parameters in the linear differential equations for x, y, and z are dependent on the areal distribution of light, there is no a priori reason why exponential decay of direct illumination differences should not be succeeded, on closing the eyes, by a series of exponentially decaying recurrent images. The values of the parameters are fixed by the conditions of the areal illumination. And this makes it also possible that the after image of a coloured light should not exhibit the complementary colour exactly. Approximately complementary images appear in the case illustrated in Fig. 28, which indicates roughly the succession of recurrent images noted by Hamaker (§ 90).

McDougall's observations on the recurrent after images of white light show that there is a tendency for green, red, and blue images to succeed each other in the order named. With moderate strength of the light, red generally appears first and is followed by green, and then the effect dies out. With stronger light, initial green is succeeded by red, blue, and green again. With still stronger light, red is at first superposed on the green ; and in very strong light blue is also superposed at first, so that the initial image is white or slightly bluish white. The superposition quickly passes, red being retarded in phase first and then blue, so that the colour changes through blue-green to green, after which the normal succession green, red, blue is followed. These occurrences are exactly of the type predicable by theory, for the values of the constants cease to be approximately constant when the limits of Fechner's law are exceeded. Thus the nine parameters vary at first, and so therefore do the phases a and a'. But variations of colour order of this kind arise in accordance with the theory even within the limits of Fechner's law. These are taken account of by the relative magnitudes of the exponential coefficients. An illustration is furnished in Figs. 29 and 30, where the

ultimate normal succession of colour impressions is not attained for some time.

Of the greatest importance from the point of view of theory is McDougall's remark that the ultimate "colours which in turn occupy the area of the after image, alone and unchanging for considerable periods, are red, green, and blue only, and these are in every case of exactly the same colour tone, although varying in brightness in different cases and in different stages of one after image. The red is a rich crimson, decidedly less orange than the red of the solar spectrum, the blue is a rich ultramarine, and the green a pure green having no inclination

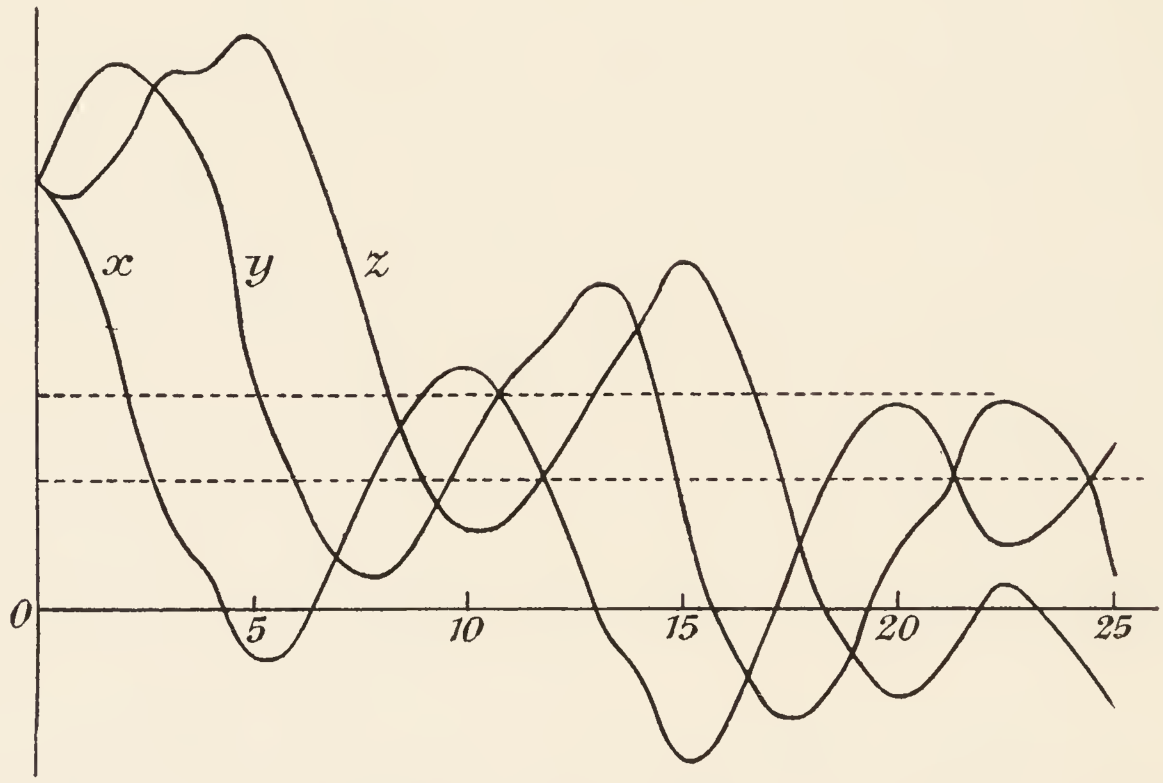

FIG. 30.—Oscillating After Images with Initial White Sensation.

towards blue or yellow." He describes the colours as intensely rich and saturated. The observations are readily made, but care is needed with regard to the conditions of fatigue.

It seems also that mental "attention," or even the exercise of "will," has influence on phenomena of the type under consideration. This, if it occurs, is merely a manifestation of the reciprocal influence of directly connected phenomena. Fatigue takes place presumably in part at the seat of transformation of the energy associated with optico-physiological action into energy associated with mental perception. Independent development of the latter might therefore produce defatigue with the consequent tendency to actual perception.

108. **Contrast Colours.**—The treatment of the subject of time effects given in the preceding sections was based upon the recognition of interconnection between locally separate parts of the retina, or even adjacent centres. So far as the formal treatment is concerned, it is of no importance where the interconnection is located. The existence of the interconnection is experimentally proved: and its existence at once leads to the conclusion that an effect of the connection must be evidenced throughout the instantaneous field of vision as well as consecutively at any one part. This simultaneous influence of neighbouring retinal illuminations is well known.

The two illuminated regions need not be near together, though their mutual influence diminishes as the distance between them increases. If the influence varies inversely as the distance, the contrast effect of an illuminated patch of given shape will be proportional to the linear dimensions of the patch. This is the law which is found to hold.

When white light is used on the patch and its surroundings also, it is found that the apparent luminosity of the patch is constant if the intensities of the lights are altered proportionately. This is a direct consequence of the presumed interconnection if its action is in accord with Fechner's law, as it practically must be if the effects are always present and Fechner's law holds generally. The reverse effect of darkening by contrast must proceed similarly.

If, further, the action be also in accordance with the linear law of Newton, as it practically must be if these cross effects are presumed to affect all vision, the instantaneous changes, due to an alteration of surrounding illumination, will be given by

$$\delta x = -\varrho_1 x' + m'y' + n'z'$$

and the similar expressions for dy and dz. The values of the coefficients can only be determined from the results of observation, but the general nature of the effect when the field surrounding a coloured patch is white can be readily seen from the consideration that the x stimulation raises the threshold value for x stimuli in the surrounding region and diminishes the threshold values for the y and z stimuli. Thus the white surroundings tend to take on the whole the aspect of the colour complementary to the active coloured light. But values of the coefficients which would otherwise give the complementary

effect are liable to influences due to fatigue, local absorption, and other causes.

With a suitable background a slight trace of colour in the patch viewed against it gives rise to a powerful contrast effect. Hering found that, when the most suitable background was used, the induced contrast was proportional to the saturation of the light on the coloured inducing area.

We shall now use x, y, z to represent the external stimuli instead of the sensation values as in the immediately preceding sections. The sensation values are then x/x_0, y/y_0, z/z_0, where x_0, y_0, z_0 are the initial threshold values. When white light whose components are x, y, z falls upon the retina from a limited patch of the field of vision, which patch is in addition illuminated by a coloured light, say z_1, the value of x_0 for the immediately surrounding region is raised in consequence of the x stimulation on the patch and is lessened in consequence of the y and z stimulations. Let the fractional elevation due to the similar stimulus be a per unit stimulus, while the fractional lowering due to the dissimilar stimulus is b per unit stimulus. The values of x_0, y_0, and z_0 on the surrounding area therefore become

$$x_0\{1+[(a-2b)(x+y+z)-bz_1]\},$$
$$y_0\{1+[(a-2b)(x+y+z)-bz_1]\},$$
$$z_0\{1+[(a-2b)(x+y+z)+az_1]\},$$

respectively, if, for simplicity merely, we presume that all the coefficients of the a type are equal, and that all those of the b type are also equal. There is thus an excess of elevation of the z threshold over the others to the proportionate extent $(a+b)z_1$. Therefore the z sensation produced by the white light which reaches the eye from the surrounding area is less than the x and y sensations produced by it. Thus the surrounding white area seems to be tinged with the colour complementary to z_1, and that to an extent which is proportionate to z_1.

We can destroy this complementary tinge by adding an amount ζ of z light to the white light which is illuminating it. If we denote the value of the square bracket above by p, the requisite condition is

$$\frac{z}{(1+p)z_0}=\frac{z+\zeta}{[1+p+(a+b)z_1]z_0},$$

which gives
$$\zeta = \frac{a+b}{1+p} z_1 z.$$

Thus ζ is proportional to z_1, and also to the white intensity, according to the theory; and its measured magnitude can be taken as a direct measure of the magnitude of the contrast colour induced by z_1. Indeed the equation could have been written down at once, for it merely expresses the fact that the requisite change of the z sensation is equal to the change of the threshold sensation which originates it.

This process was introduced by Pretori and Sachs who employed the method of rotating sectors. The resultant illumination of the inner and outer radial thirds of the disc was the same, and the two constituted the patch spoken of above. The results of their observations verify the deduction given above.

Let now, by decrease of black sectors in the inner and outer thirds, the amount of white be kept constant while z_1 is increased by z'_1. This may be compensated, as regards the fresh contrast aroused in the middle third through increase of ζ, by the amount
$$\zeta' = \frac{a+b}{1+p} z'_1.$$

But, alternatively, we may remove the added contrast by a reduction of the white in the middle area. In fact, writing
$$(a-2b)(x+y+z) - b(z_1 + z'_1) = p',$$
the conditions of the fresh contrast give for the x, y, and z stimuli on the retina from the middle area the relations
$$\frac{x}{(1+p')x_0} = \frac{y}{(1+p')y_0} \neq \frac{z+\zeta}{[1+p'+(a+b)z_1]z_0}.$$

So, instead of adding ζ' to ζ we may subtract, by increase of black sectors, from the x, y and z stimuli respectively, the quantities ξ', η', ζ', in order to reduce the amount of white. This gives
$$\frac{z-\zeta'}{(1+p')z_0} = \frac{(z-\zeta')+\zeta}{[1+p'+(a+b)z_1]z_0}.$$

Hence
$$\zeta' = \frac{p-p'}{1+p} z.$$

Thus a proportionate diminution of white in the middle area annuls the contrast produced by a given increase of colour in the contiguous areas, while the white there is unaltered. Here, again, the equation merely asserts that the change of sensation induced through alteration of a threshold value can be annulled by a proportionate change in the direct stimulus.

The general results of colour contrast follow from the application of Fechner's law to the postulated linear interactions accompanying stimulation of adjacent retinal areas. There lies herein a further justification of the extension of that law, by Helmholtz, to the three independent constituents of a compound colour sensation.

Other results obtained by Pretori and Sachs follow with equal ease. One is that the contrast induced by an increase of white in the inner and outer areas can be annulled by a proportionate increase of white in the middle area when no change is made in the amounts of colour in the three regions. Another, not quite so invariably verified, is that proportionate increase of the white and the coloured components of the inner and outer areas produces no change of contrast in the middle area. In the exceptional cases, the contrast increased nearly in proportion up to a definite maximum. Conversely, with constancy of the inner and outer areal conditions, increase of white in the middle region caused increase of contrast up to a definite maximum.

The equations are readily applicable to the explanation of the calling out of a coloured sensation, previously below the threshold, by means of a feeble white stimulus.

In a complete discussion, the reaction from the middle region upon the inner and outer regions has to be taken account of. The question of the equality of the quantities of type a, as also of those of type b, amongst themselves respectively, necessarily remains ; and the limits of applicability of Fechner's law also enter.

CHAPTER XIV

THE PROBLEM OF THE MECHANISM

109. **Areal Distribution of Visual Characteristics.**—In preceding chapters the variation of peculiarities of vision according as light is incident upon one or another region of the retina has been to some extent considered. The facts that perception is associated with the rod and cone layer, and that the areal distributions of rods and cones are entirely unlike, would, on the ground of structure alone, lead to the expectation that visual characteristics might correspond in some manner to the mode of distribution of these organs. And the further facts that, in regions where rods are absent, the cones are more rod-like, while they are least rod-like in regions where rods preponderate, would not much modify the expectation. But there is no a priori reason why cone action should be essentially different from rod action in the phenomena of excitation of perception. Observation or experiment alone can settle the question. And the result of the appeal is to bring into evidence very characteristic differences.

In the foveal region, which subtends a diametral angle of about 1°, rods are entirely absent, and discrimination of form and colour is most complete. It is in that region that freedom of the rod and cone layer from overlying structures is greatest. The absence of rods persists throughout an area subtending a diametral angle of fully 3°.

The appreciation of colour difference becomes more and more defective as the area of stimulation is situated more and more eccentrically. Violet first ceases to be distinguishable from colourless light, and, under more moderate illumination, the other main colours disappear in the order green, red, yellow, blue.

Finally, a monochromatic or achromatic region exists in the

neighbourhood of the retinal periphery. The locus upon the retina of the boundary beyond which a particular colour is not appreciated is very variable. It depends upon the size of the stimulated area, upon the intensity of the incident light, on its saturation, and on the condition of adaptation induced by precedent illumination.

Colour may disappear even at the fovea when the illumination is sufficiently feeble; and there is ultimately a lower limit of intensity at which all sensation of light vanishes. There is thus a chromatic interval; but, with a sufficiently small area of stimulation, colour and luminosity disappear simultaneously (§ 111). The magnitude of the interval is affected by adaptation if the intensity has given rise to appreciable fatigue. Under normal conditions there is comparatively little adaptation. Burch found that the rod-free region could exhibit colour as long as luminosity was manifest, provided that extreme, though slow, recovery from fatigue takes place in consequence of the entire exclusion of light for several hours. The fovea itself is practically night blind (§ 86).

The dependence of colour appreciation upon intensity of illumination is evident even in the peripheral parts of the retina. Under very strong illumination colour may be perceived in at least the close neighbourhood of the periphery if the saturation is great and the area of stimulation is large enough.

Another important point is that the retinal boundary for colour appreciation is generally not the same for complementary spectrum colours. Abney found that red light of wavelength $650\mu\mu$ disappeared at less obliquity than that which was needed to cause disappearance of the complementary amount of light of wavelength $500{\cdot}2\mu\mu$. Similarly the normal order of yellow and blue was inverted in the case of a yellow green of wavelength $561{\cdot}4\mu\mu$, and a blue of wavelength $460{\cdot}3\mu\mu$. The boundary for the latter coincided with that for the green of wavelength $500{\cdot}2\mu\mu$.

Nevertheless, it is found to be possible to have complementary colours which, under suitable conditions of saturation, intensity, and area of stimulation, do not change colour as their place of incidence recedes from a central to a peripheral locality. The red, so found by Hess, was bluer than the red of the spectrum; the green was that of wavelength $495\mu\mu$; and the yellow and

blue were of wavelengths $574 \cdot 5\mu\mu$ and $471\mu\mu$ respectively. All spectrum colours of greater wavelength than $549\mu\mu$ tend to approximate to the invariable yellow as the periphery is approached ; all of less wavelength tend towards the invariable blue. Baird found that the condition of coincidence of disappearance of these complementary pairs of colours is maintained through dark adaptation and light adaptation alike. This supplies a common characteristic of the dichromatic vision to which all normal eyes are subject when the retinal area of stimulation lies between the loci of disappearance of the two invariable pairs of complementary colours.

On the Young-Helmholtz theory this fact points strongly to the conclusion that the "red green" colour blindness of that zone is due to failure of the red fundamental, not absolutely, but, through its incorporation with the green fundamental, so that both fundamental activities are called forth necessarily in the constant proportion which gives yellow-blue vision. This is in accordance with the facts of one-eyed colour vision. Towards the outer parts of the yellow-blue zone fusion proceeds further, till, in the peripheral parts, white, or perhaps bluish white, vision alone remains. It is specially noticeable that the bluish red colour of the invariable red agrees in type with that of Helmholtz's ultimately deduced red fundamental (§ 77), and that the blue and green agree respectively in type with his ultramarine and yellowish-green fundamentals.

In dark adaptation of the fovea, binocular summation of effects takes place. This differs from the normal result when light adaptation is carefully maintained. In this case binocular effects do not add, nor do effects of separate stimuli sum. The periphery is more sensitive to flicker than the fovea, if it be unfatigued. But it quickly fatigues and is then less sensitive than the fovea. With weak coloured illumination the fovea is more sensitive to red flicker and the periphery to blue flicker.

110. **Areal Distribution of Threshold Values.**—In dim vision the stimulus is weak and the threshold value is low when adaptation has taken place. In bright vision the stimulus is strong and the threshold value is high when adaptation is complete. When the dimness is great, colour disappears. This is most simply explained by the nature of the connections of adjacent centres. The analogy of these actions, and the

gradually accumulating or even oscillating effects already considered, to the effects of disturbance of fluid equilibrium in suitably interconnected vessels containing liquid, is, in so far as the mathematical form of the relationships is concerned, very obvious.

When a coloured light, sufficiently strong to produce a steady impression of colour, falls upon the retina for a sufficiently short time, no impression of colour is produced. The appearance is that of white light. Zahn compared this white with a surrounding steady white, and so estimated the luminosity requisite for the initiation of a colour impression. These are called the "minimum time" luminosities. The results so obtained are in entire concordance with those got by the minimum field method. All methods, when applied to the periphery, show that the threshold value has its minimum, and therefore that the luminosity of the colourless but bright peripheral spectrum has its maximum, in the region of yellow wavelengths. This is entirely different from the condition in the colourless spectrum of dim-vision in the central but non-foveal part of the retina, wherein the maximum luminosity lies amongst the green wavelengths.

The latter feature is of great importance. And the fact that the initial sensation given by any coloured light is that of white is also noteworthy from the point of view of the analogy given above.

The foveal colourless luminosity curve is also found to have its maximum at yellow wavelengths; so that it agrees with the normal peripheral value as obtained in good illumination. The curves of the chromatic and luminosity thresholds for the fovea differ from the corresponding normal central curves chiefly in the continuous diminution of the threshold values towards the longer wavelengths.

In the cases of dichromasy characterized by defective green sensation, the bright luminosity curves and the ordinary peripheral luminosity curves alike have their maxima in the green. In the bright luminosity curves and the ordinary peripheral luminosity curves of dichromatic eyes which are defective in the red sensation, the maxima lie in the yellow, and so agree in this feature with the ordinary, or bright, peripheral luminosity curve of the normal eye. The determination

of peripheral curves in dim light is very uncertain. In red defective and green defective dichromasy alike the dim-vision luminosity curve has its maximum in the green, shifting thereto in the latter case as the luminosity lessens. In this respect these agree with normal vision. In cases of monochromasy also the maximum is in the green.

We have just seen that the nature of the normal dichromasy of the intermediate retinal region indicates fusion of the red and green sensations. On this presumption, the location of the peripheral maximum of luminosity in the yellow could be anticipated.

The chromatic interval is greater at the periphery than at the central parts.

111. **Dependence of the Threshold Values on the Area of Stimulation.**—Abney carried out experiments to determine how the threshold value varies when the area of the retina which is under stimulation is altered. He found that, for different spectrum colours, the diagram obtained by plotting the logarithms of the intensities as ordinates against the diameter of the illuminating aperture expressed as powers of 2 consisted of a set of straight lines practically parallel to each other. The same result held in the case of extinction of luminosity, the slopes of the two sets of lines, however, being different. The slopes were such as to indicate the existence of a size of aperture, and therefore area of stimulation, with which colour and luminosity could disappear together. This aperture was largest in the case of red light.

The results can be expressed algebraically by the equation

$$ {}_rI_0 = A\left(\frac{2}{r}\right)^k, $$

where r is the radius of the aperture, k is the constant slope of the lines in the diagram, A is a constant which has different values for the two sets of lines, and ${}_rI_0$ is the threshold intensity with the aperture r. The value of k in the case of the colour threshold, was unity, and, in the case of the absolute threshold, it was 2/3. When the angular diameter of the area exceeds 4°, the threshold value is independent of r.

It follows that, in the case of colour, the product of the threshold intensity into the square root of the area of illumina-

tion is constant. This identical law is found to hold, in the case of dark adapted peripheral areas, for the luminosity threshold provided that the area has a diameter within the limits 1° and 10°.

In the case of the absolute threshold, the intensity does not vary quite so rapidly as the aperture increases.

Riccò showed that, when the foveal field has the minimum area which can produce sensation, the threshold intensity varies inversely as the area of the aperture. This result has been verified, and is of great importance, for it indicates that a constant total stimulus is requisite for the production of the smallest foveal sensation.

112. **Effect of the Area of Stimulation on the Extent of Field.**—Charpentier showed that the extent of the field sensitive to white light was unaffected if the intensity varied inversely as the square root of the area, provided that the angular diameter did not exceed about two-thirds of a degree. When the diameter exceeded that value, the intensity varied inversely as the area, that is, the extent of the field is constant if the total stimulus is constant.

Coloured fields being used with partial dark adaptation, Abney found that successive diminutions of the linear dimensions of the aperture in a constant ratio caused successive constant diminutions in the linear dimensions of the field. Thus the former increases in geometrical progression as the latter increases linearly. In symbols the law may be written

$$r = r_0 \varepsilon^{p\vartheta},$$

where p is constant for a given direction of the field, and r_0 is the radius of the minimum area of stimulation with the given intensity of the light. The applicability of the law is subject to the condition that the angular diameter of the stimulated area shall not lie outside the limits of 10′ and 4°28′.

113. **Effect of Intensity on the Extent of the Field.**—Abney showed that successive diminutions of the linear dimensions of intensity in a constant ratio gave rise to successive equal diminutions of the angular diameter of the field of vision. This may be expressed by the condition

$${}_{\theta}\mathrm{I}_{0} = {}_{0}\mathrm{I}_{0} \varepsilon^{q\theta},$$

where q is a constant for all wavelengths, ${}_0I_0$ is a constant for any one wavelength, and ${}_\theta I_0$ is the intensity required for the field θ. The value of ${}_0I_0$ is less for green light ($530\mu\mu$) than for red (670·5), yellow (589·2), and blue (430·3). The values for these increase in the order given.

114. **Bearing on Visual Characteristics: (1) Interconnection of Stimuli.**—The fact that in foveal illumination the threshold intensity is inversely proportional to the area of illumination shows that, in all parts of that region, contribution is made in one mode towards the production of sensation. With a given total amount of illumination, just sufficient to originate sensation, the density of its distribution per unit area of the fovea may follow any law. The threshold value at each point is just equal to the local intensity there. If the local stimulation were entirely independent of surrounding stimulation and itself induced equilibrating fatigue, there should be no fixed value of the total illumination which would just arouse sensation. There must be cross influences amongst the several parts of the illuminated mechanism, wheresoever the seat of that interaction may lie. Part of the action must be direct and the remaining part be induced, so that the whole system is in self equilibration, and the threshold value is just maintained in equality to the external intensity.

In retinal regions wherein, under the conditions specified in § 111, the threshold value varies in inverse proportion to a power of the linear dimensions of the area of stimulation, the threshold value does not fall so rapidly as the area increases. In the case of the sensation of luminosity, when $k=1$, it only falls in proportion to the linear magnitude of the area; and, in the case of colour, when k is less than unity, the rate of decrease of the threshold as the area increases is still slower. When the extent of the area exceeds that of the rod-free region by less than one degree in angular diameter, the threshold intensity ceases to vary with r. The change is progressively in the direction of average independence of the partial areas of each other. If the independence is not complete in actuality, there must be balancing of fatigue and defatigue.

Independent confirmation is given by Abney from measurements of the apparent luminosity of spectrum lights when the aperture is increased while the actual intensity is kept constant.

The threshold being exceeded, the apparent intensity increases when the aperture is increased, though it does not increase in proportion. Thus mutual defatiguing influence from adjacent parts is made evident.

The defatiguing effect is greatest at the fovea, and diminishes as more remote regions are included, until, when an area of about 4° in radius is exceeded, the effect ceases. And, in the peripheral monochromatic or achromatic region, a defatiguing effect is again evident.

In this connection the following remark by Abney is noteworthy : "The light from a square, or a disc, or an oblong, just before extinction, is a fuzzy patch of grey, and appears finally to depart almost as a point. This can scarcely account for the smallest width of an illuminated surface determining the intensity of the light just not visible ; but it tells us that the light is still exercising some kind of stimulus on the visual apparatus, even when all sensation of light is gone from the outer portions. The fact that the disappearance of the image takes place in the same manner whether viewed centrally or excentrally tells us that this has nothing to do with the yellow spot, or fovea, but is probably due to a radiation of sensation (if it may be so called) in every direction on the retinal surface. Supposing some part of the stimulus impressed on one retinal element did radiate in all directions over the surface of the retina, the effect would be greatest in the immediate neighbourhood, and would be inappreciable at a small distance, but the influence exerted upon an adjacent element might depend not only on its distance, but also upon whether it was or was not itself excited independently. Following the matter out further we should eventually arrive at the centre of an area as the part which was the recipient of the greatest amount of the radiated stimuli, and consequently that would be the last to disappear. With a slit aperture the slit is visible till extinction is very nearly executed, but it finally merges into a fuzzy spot at the moment before it finally fails to make any impression of light."

The radiation of stimuli here spoken of is the phenomenon described in the foregoing treatment as a cross influence between distinct centres of stimulation. With a cross influence dependent in some manner inversely upon the distance separat-

ing the centres, and giving rise to defatigue, the point at which defatigue is most evident, e.g., the centre of an illuminated disc, is the point at which decreasing illumination would be last perceived.

115. **Bearing on Visual Characteristics : (2) Type of Action.**—In its mathematical relationships the whole question of the particular sensation which is called forth by stimulation of a definite area, or set of areas, of the retina, is merely that of the form of the functions r_0, g_0, b_0 regarded as dependent on intensity, area of stimulation, the areal co-ordinates, and the instantaneous values of the effects of precedent illumination or the self light of the eye. This is a sufficiently complicated problem, greatly in need of experimental exploration, to which the work briefly sketched above forms a worthy beginning.

In so far as the results hitherto obtained are sufficiently precise, it appears that, with gradually lessened intensity of stimulation, foveal sensation steadily approximates to that of white light, and ultimately coincides therewith. That is to say, in feeble illumination, the three fundamental threshold values respectively acquire magnitudes which bear a common ratio to the instantaneous values of the three fundamental stimuli. When this stage is first reached, we say that the threshold value for colour has just been attained. But there is no a priori reason why a colour threshold should not be lowered by a process of defatigue or adaptation. Colour, in that event, could reappear at a lower intensity. This is what Burch observed (§ 109). While the sensation is submerged, the threshold activity still proceeds. Many phenomena show this ; for example, the power of colourless green to excite a coloured after image in a colour-blind eye (§ 100). Another example occurs in the case of normal vision in the retinal region lying between the boundaries at which the colours of two complementary lights cease respectively to be perceived. In that region white light gives a white sensation. Therefore the non-effective colour is still effective as a complementary.

But, while the colour fatigue is maintained at a high value, the threshold fatigue is also necessarily high, and foveal sensation normally vanishes under relatively good illumination. In this condition the colourless spectrum has its maximum of intensity in the yellow region. The three equal sensation

values, r/r_0, g/g_0, b/b_0, are called out most strongly by yellow light.

When we pass from the fovea to the surrounding rod-free area, the conditions of perception alter, and the change proceeds more markedly when the rods also are affected. But the evidence is clear that the illumination of one region affects another ; and the interconnection remains, although the cross transmitted stimulus does not rise to the threshold value. Now the maintenance of the equality of action and reaction in the transference or transformation of energy requires a reciprocal action, from even unperceived stimulation, upon the threshold values at the directly-illuminated parts of the retina : which reaction might affect the perception there. Thus, as we pass to non-central regions, we might expect to find a gradual change in the laws of perception, such as those mentioned in the immediately preceding sections. The changed interactions, or the changed conditions of the receptive organs, as a somewhat different type of organ is gradually introduced, give rise at definite stages to the result that definite wavelengths of light cause equal sensation values of the three fundamental activities, and so produce a colourless sensation. Finally, this condition spreads throughout the entire spectrum in the immediate neighbourhood of the periphery, after passing through a stage in which yellow-blue dichromasy is apparent. When the colourless stage is reached, the maximum of luminosity is at the yellow part of the spectrum, as already noted.

In dichromatic vision of the red deficient type, the maximum of luminosity, both in central and in peripheral vision, is shifted towards the green wavelengths from the normal location in the yellow. With green deficiency, the change of the maximum is towards longer wavelengths. In general the difference between the central and the peripheral luminosities at the various wavelengths in red deficiency is of the same nature as in normal vision. It is of the opposite nature in green deficiency. In all colour types of vision, the ultimate change as luminosity decreases gives the maximum in green light. And, in monochromatic vision, the maximum is always in the green. Thus deficiency in green perception is rather less different from the normal type. Whether the maximum changes from the yellow

to the green in peripheral vision when the intensity is extremely reduced is difficult to determine ; and the high values of the foveal thresholds put the effects of great reduction of intensity outside the possibility of direct observation usually.

All these conditions point towards a common type of action at all parts of the retina. There is no invariable condition at any one part. Under some conditions colour blindness may be evident, and, under others, such as greater intensity, or greater area, the vision may be trichromatic. The same feature occurs in dichromatic vision. The variations of the threshold values take account of all these aspects. That there should be some difference between cone vision and rod vision is only to be expected in consequence of the structural difference. And a functional difference is made evident through the differences of bright and dim vision, and of foveal and peripheral vision. But it scarcely appears that there is any essential qualitative difference between the actions of the two organs. In feeble light the cones give colourless vision, and in strong light the rods give coloured vision. On the whole their functions seem to be respectively those of response to strong and to weak stimulations. Too large a luminous sensation is associated with the sensation of pain and the possibility of structural or functional injury ; and it is provided against by automatic closing of the iris or of the eye. A corresponding provision is that of automatic increase of the thresholds by fatigue, and high threshold values are normally associated with the cone vision of the fovea, which is used in direct regard of a luminous object. Low threshold values, on the contrary, are normally associated with the rod vision of the more peripheral parts of the retina, in consequence apparently of the large amount of the visual purple which is present : and these are of value in dim vision in which width of field is of most importance. In dim light the iris automatically opens and so supplies increased illumination to the peripheral parts. Automatically, also, the thresholds fall and give large sensation from small stimulus. It is in accordance with these provisions that the foveal cones should be specially discriminative of form, which is desirable in direct scrutiny, while the peripheral rods are discriminative of motions throughout a wide field. It may be possible that the provision which ensures these respective properties may be

the condition which determines the quantitative differences in colour perception.

Nevertheless, the view that the rods are efficient in perception of luminosity only while the cones are responsive to colour stimulus, which is the basis of the Duplicity Theory of v. Kries, is not antagonistic to the trichromatic theory of Young and Helmholtz. It merely limits the mode of application of the latter by raising the question of structure.

116. **Relation to Subsidiary Hypotheses.**—The attitude of both Young and Helmholtz towards subsidiary assumptions was very clearly expressed. These were merely illustrative of the essential formal laws. Even the postulate of three sets of nerve fibres was only tentative. Given the formal laws as determined by the strict processes of physical science, the trichromatic theory seeks only to determine their formal consequences which are then to be compared with the results of observation and experiment. The determination of the mechanism is a matter for the anatomist and the physiologist alone to investigate, though their work may be aided by the work of the physicist and the psychologist ; for it is a vain thing to look for structure or function in a direction which is necessarily incompatible with ascertained law. Though these workers work independently their results must ultimately coincide.

The older view of the origin of dichromasy, as stated by Young and Helmholtz, implied that normal vision and the, at most three, different types of dichromatic vision constituted, on the whole, entirely separate types. But the newer view, introduced by Helmholtz when the cases of one-eyed yellow-blue vision showed that the simpler assumption was not sufficiently wide, is wide enough to give a field of continuous variation ; and is quite possibly, because of the nature of the actual mechanism of vision, wider than is necessary to account for the actual facts. The law of distribution of types, when it is found, can neither confirm nor oppose the theory. It can at most lay down restrictions upon it. Dr. Houston's recent work is forming a basis for the enunciation of the law.

There is no absolute reason why one worker should not postulate a seven-fold mechanism, or even one of much greater complexity, so long as results which do not involve the

independence of the several units are not contemplated. The worker with twenty units could state his results in terms of these; but the knowledge of the universe thereby conveyed is less complete than it might be, for trichromasy is a proved fact and no longer a postulate. Seventeen equations connecting the twenty units have to be expressed before, in a cumbrous way, the fulness of the information conveyed by the simple three-co-ordinate treatment is equalled.

Helmholtz's illustration of action through three different photochemical substances suits the main phenomena, and may even be reconcilable with a single set of connections to a triple set of cerebral centres. This would be the simplest view, seeing that certain phenomena, such as those of binocular vision, prove the existence of cerebral interconnections in any case. Such speculations are of use as tentative guides, though they may or may not indicate the actual paths.

The fundamental laws of the trichromatic theory form the high mountain ranges which dominate and compel the paths. The great instinctive pioneers read the compulsion aright from perception of the peaks, and it is rarely that they made mistakes. It is of the most intense interest to note how perceptively Newton, and Young, and Helmholtz walked in the unknown land.

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